Central Bank Policy Responses to Volatile Capital Flows[*]

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Abstract Emerging market economies frequently experience volatile capital flows due to large foreign interest rate shocks, so it is natural to ask how central banks should react when external volatility is high. Using data from emerging market economies, I develop a Markov-switching dynamic stochastic general equilibrium model that features external shocks of time-varying volatilities to study their monetary policy choices. I find that by allowing the response coefficients in the interest rate rule to vary according to the external volatility regime, the central bank can improve welfare while maintaining macroeconomic stability. The optimal simple rule suggests that the central bank should target inflation when external volatility is low and stabilize exchange rates when it is high, which is akin to the "leaning against the wind" approach adopted by some emerging market central banks.

Keywords: Macroprudential policy, emerging markets, central banking, capital flows, Markov-switching models.

JEL Classification Numbers: E52, F32, F41, G15.

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1. Introduction

Central banks in emerging markets have followed two major trends in policy reform over the past few decades: liberalizing their capital accounts and adopting a flexible exchange-rate regime. While emerging market economies continue to impose more rigorous capital account regulations compared with advanced economies, capital account openness has increased substantially.[1] In the meantime, cross-border capital flows have risen dramatically (Lane and Milesi-Ferretti, 2007) and the fixed exchange-rate regime was abandoned in favor of a flexible exchange-rate regime. Instead of pegging their currencies to the dollar, many emerging market central banks are now adopting inflation targeting as the foundation of their monetary policies (Roger, 2010).

However, having open capital accounts also makes emerging market economies more vulnerable to large foreign interest rate shocks, often resulting in volatile capital flows and considerable exchange-rate fluctuations. I construct the real foreign interest rate processes for eight emerging market economies and find strong evidence of time-varying volatilities. The foreign interest rate process is characterized by two regimes of shock volatility: the high-volatility regime and the low-volatility regime. Output and foreign exchange rates are much more volatile under the high-volatility regime.

Thus, it is natural to ask how central banks should react when external volatility is high. While recent studies suggest that temporary central bank intervention can be desirable (Farhi and Werning, 2014), many studies favor an inflation-targeting interest rate rule (Galí and Monacelli, 2005), not to mention warning about causing macroeconomic instability if the central bank chooses to respond discretionarily. This divide among academic scholars has also influenced emerging market central banks–some of them favor temporary intervention, often referred to as "leaning against the wind," while others are strict inflation targeters (e.g. South Africa).

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[1] See Chinn and Ito (2006) for more information about the capital account openness index they construct, which is available at: `http://web.pdx.edu/~ito/Chinn-Ito_website.htm`

To answer this question, I develop a Markov-switching dynamic stochastic general equilibrium (MS-DSGE) model that features external shocks of time-varying volatilities to study the monetary policy choices of central banks in emerging markets. Driven by larger foreign interest rate shocks, an economy fluctuates to a greater extent under the high-volatility regime. The central bank adopts an interest rate rule that responds to inflation, the output gap, and exchange-rate fluctuations, in which the response coefficients can vary under differing volatility regimes. Thus, the question becomes whether allowing this dimension of flexibility can improve welfare without causing macroeconomic instability.

One major contribution of this paper is that it is the first to examine whether regime-switching monetary policy rules can ensure macroeconomic stability. I solve the MS-DSGE model using an algorithm from Farmer, Waggoner, and Zha (2011) and estimate it using Thailand data from 2001Q1 to 2015Q2. I examine a wide range of response coefficients in the interest rate rule and find that regime-switching interest rate rules are unlikely to cause macroeconomic instability.

In addition, I find that temporary intervention is effective in terms of stabilizing the economy in highly volatile periods. I characterize the "leaning against the wind" approach used by emerging market central banks as a regime-switching interest rate rule that responds to exchange rate fluctuations more aggressively under the high-volatility regime. I compare its performance with a constant-coefficient inflation-targeting rule. The impulse responses indicate that output and consumption fluctuate to a lesser extent under the former rule when the economy is hit by a large foreign interest rate shock.

Furthermore, welfare analysis shows that the optimal simple rule contains some "leaning against the wind" features. The optimal simple rule features no response to exchange-rate fluctuations under the low-volatility regime and a strong response under the high-volatility regime. This is because, under the low-volatility regime, the existence of productivity shocks and foreign demand shocks creates trade-offs between stabilizing inflation and the exchange rate, so the optimal operating rule should not respond to the exchange rate. Under the high-volatility regime, foreign interest rate shocks become the most important source of disturbance so monetary policy should respond to exchange rate fluctuations aggressively.

Finally, my research makes an additional contribution in studying the effect of temporary capital controls by allowing the central bank to adopt capital controls depending on the external volatility regime. Capital control is modeled as a quadratic portfolio adjustment cost, which reduces capital-flow volatility by inserting a wedge in the intertemporal Euler equation. It turns out that capital controls can effectively stabilize aggregate output and consumption in the short run but are welfare-reducing in the long run because households, anticipating possible capital controls in the future, will behave inefficiently under such circumstances.

The rest of the paper is organized as follows. Section 2 reviews the related literature on open economy macroeconomics and emerging market economies. Section 3 studies foreign interest rates and empirically assesses the regime-switching assumption. Section 4 develops the MS-DSGE model used to study the problem as well as the solution algorithm. Section 5 summarizes the data source and the estimation results. Section 6 explains the main results of the research and section 7 concludes.

2. Related literature

This research builds upon two existing strands of literature: studies on the analytical framework of the new open economy macroeconomics and studies on emerging market economies. In this paper, I try to link the existing analytical framework with emerging market characteristics in order to achieve a better understanding of emerging market central banks and their policy choices.

The new open economy macroeconomics literature serves as the modeling foundation. Recent examples of the dynamic stochastic general equilibrium (DSGE) framework I employ in the paper can be found in Clarida, Galí, and Gertler (2002) and Devereux, Lane, and Xu

(2006). In particular, I adapt the modeling framework of Galí and Monacelli (2005), who develop a small open economy model with nominal rigidities. They suggest that it is optimal for a small open economy central bank to target domestic inflation and the output gap.

More recently, there have been some attempts to incorporate more emerging market features into the existing analytical framework. For instance, Aguiar and Gopinath (2007) suggest that emerging market economies face large nonstationary productivity shocks so that growth trend fluctuations constitute their business cycles. García-Cicco, Pancrazi, and Uribe (2010) use a small open economy with financial frictions to characterize business cycles in emerging markets.Liu and Spiegel (2015) study optimal monetary policy and capital account restrictions in a small open economy. Anand, Prasad, and Zhang (2015) find that a developing country's central bank should target headline inflation instead of core inflation.

Conceptually, the nature of many emerging market economies suggests that their central banks should not operate entirely based on the experience of advanced economies. For example, emerging market economies face prevalent foreign interest shocks and country-specific risk premium shocks, which are particularly important in the era after the global financial crisis due to unconventional monetary policy. Uribe and Yue (2006) find that interest rate shocks represent an important driver of business cycles in emerging countries, accounting for 30 percent to 42 percent of the variance in output. If emerging market central banks are reluctant to "import" monetary policy from abroad, they have to decide whether to adopt a flexible exchange-rate regime or impose capital controls, and a recent strand of research suggests that capital controls are necessary regardless of the exchange-rate regime (Rey, 2015). Edwards (2015) also finds similar results based on data from Latin America.

Another policy feature in emerging market economies is that capital controls are much more prevalent. One important reason that prudential capital controls are necessary is that households face pecuniary externalities. Bianchi (2011) develops a DSGE model with an occasionally binding collateral constraint and shows that macroprudential policy limits overborrowing and improves welfare. Similarly, Korinek (2011) differentiates prudential capital controls from traditional capital controls and argues that the former reduce the risk of financial crises. Apart from pecuniary externalities, there are alternative channels that make prudential capital controls appealing. Schmitt-Grohé and Uribe (2012) find that prudential capital controls are helpful for peggers because of downward nominal wage rigidity. Farhi and Werning (2014) prove that capital controls are welfare-improving even under a flexible exchange-rate regime because they smooth intertemporal terms of trade.

3. Foreign Interest Rate Shocks

Since foreign interest rate shocks are a major driving force of business cycles in emerging market economies and are directly responsible for volatile capital flows, I start by characterizing the foreign interest rate process. I follow Uribe and Yue (2006) and compute the real foreign interest rate for an emerging market economy by adding the real US interest rate and its country spread together.

I compute the real foreign interest rates for eight emerging market economies: Argentina, Brazil, Ecuador, Malaysia, Mexico, South Africa, Thailand, and Uruguay. The real US interest rate is derived from the T-bill rate and inflation while the country spread is retrieved from the JP Morgan Emerging Market Bond Index (EMBI). For all emerging market economies in the sample, country spreads account for a major share of the variations in foreign interest rates.

Since the foreign interest rate is assumed to be exogenous in my model and exhibits time-varying volatilities, I can estimate it separately using a regime-switching model.[2] I estimate the real foreign interest rate processes for the eight emerging market economies individually using a univariate AR(1) model with unobserved regime switches.

$$r_t = (1 - \rho_{r,s_t})\bar{r} + \rho_{r,s_t} r_{t-1} + \varepsilon_{t,s_t}, \quad \varepsilon_{t,s_t} \sim N(0, {\sigma^{r}_{s_t}}^{2}) \tag{1}$$

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[2] Hamilton (1989) introduces regime-switching models into economics and I follow his algorithm when estimating the real interest rate processes.

Table 1. Estimates of Foreign Interest Rate Processes

Country	ρ_r	σ_L^r	σ_H^r	p_L	p_H
Argentina	0.90	0.29	3.34	0.95	0.76
Brazil	0.94	0.14	0.81	0.94	0.83
Ecuador	0.83	0.28	2.36	0.93	0.76
Malaysia	0.93	0.11	0.44	0.96	0.80
Mexico	0.91	0.13	0.85	0.98	0.82
South Africa	0.95	0.07	0.22	0.84	0.85
Thailand	0.97	0.08	0.33	0.92	0.82
Uruguay	0.94	0.12	0.74	0.93	0.73

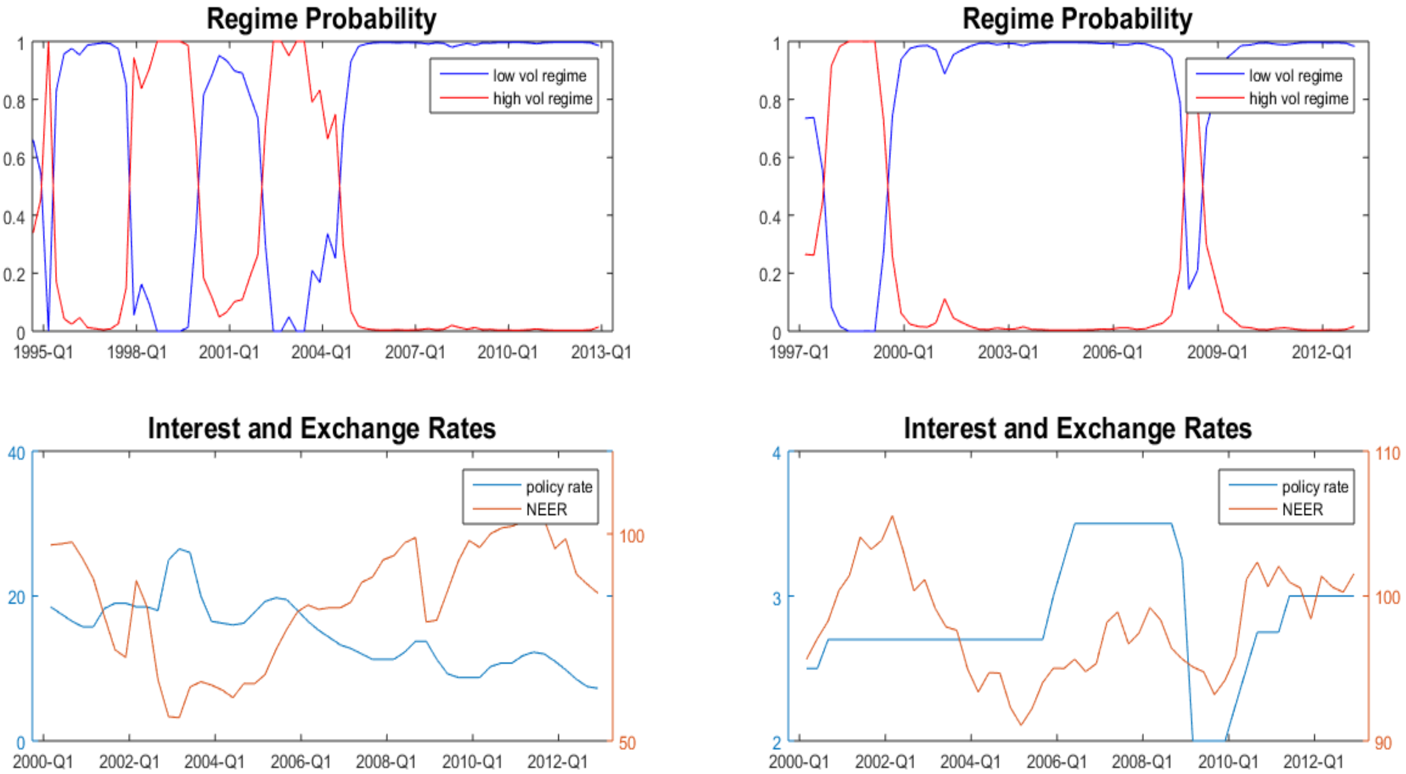
Notes: Real interest rates are computed by adding US interest rates and country spreads together. Data availability depends on individual countries. ρ_r is the constant autoregressive coefficient. σ_L^r and σ_H^R are the shock volatility under the low- and high-volatility regimes, respectively. p_L and p_H are the probabilities of remaining in the low- or high-volatility regime in the next period, respectively.

For all eight foreign interest rate series, the regime-switching estimation detects two regimes, under which shock volatility differs significantly. The autoregressive coefficients are found to be constant across the two regimes, so foreign interest rates can be modeled as AR(1) processes with time-varying volatility.

The parameter estimates are very similar across the eight emerging market economies and the results are presented in Table 1. The median probabilities of remaining in the low- or high-volatility regime in the next period are 0.95 and 0.80, respectively. The median standard deviations of foreign interest rate shocks are around 0.1 and 0.5 under the low- or high-volatility regimes, respectively.

Figure 1 visualizes the volatile regime switches in foreign interest rates and the corresponding central bank policy responses. I plot the probability of volatility regimes, the nominal effective exchange rates (NEER), and the policy rates separately for Brazil and Malaysia. As we can see, each country experiences several switches between the low- and high-volatility regimes. For example, Brazil encountered substantially higher foreign interest

Figure 1: Volatility Regime Probabilities and Policy Rate Responses



(a) Brazil

(b) Malaysia

Notes: In the regime probability plot, the red (blue) line represents the probability that an economy is under the high- (low-) volatility regime. The probabilities are estimated using the regime-switching model in equation (1) and at any time, the two probabilities add up to 1. In the interest and exchange rates plot, the blue line is the central bank policy rate and the red line is the nominal effective exchange rate (NEER) of a country.

rates in 2002 and 2003 due to an increase in country spreads, accompanied by considerable currency depreciation. On the other hand, foreign interest rates dropped significantly after the federal reserve adopted unconventional monetary policy.

Notably, exchange rate fluctuations are larger under the high-volatility regime and policy rates also appear to be more responsive to foreign exchange rate fluctuations. This is consistent with the anecdotal evidence that emerging market central banks prefer "leaning against the wind" when external volatility is high.

4. Model

To study the implications of adopting regime-switching monetary policy rules in emerging market economies, I develop a small open economy model with nominal rigidities and foreign interest rates of time-varying volatilities. In contrast to the simplifying assumption of complete markets in Galí and Monacelli (2005), the home economy can access the international financial market only through risk-free borrowing and lending, which is subject to foreign interest rate shocks.

Foreign interest rates are characterized by a Markov process with two underlying regimes: the high-volatility regime and the low-volatility regime. Under the high-volatility regime, foreign interest rate shocks are larger whereas under the low-volatility regime they are smaller. Consequently, capital flows are more volatile under the high-volatility regime. The paper then studies whether central banks should intervene during periods of volatile capital flows and, if so, what policy instruments they should use.

4.1. Households

The home economy consists of a large number of identical and infinitely-lived households that consume both domestically produced goods and foreign imports. The representative household maximizes the discounted stream of utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \tag{2}$$

where $\beta \in (0,1)$ is the discount factor, C_t is the composite consumption of the representative household in period t, including home and foreign goods, and N_t is the labor supplied by the representative household. The utility function takes the form:

$$U(C_t, N_t) = \left(\frac{{C_t}^{1-\sigma}}{1-\sigma} - \psi \frac{{N_t}^{1+\phi}}{1+\phi} \right) \tag{3}$$

where σ is the risk-aversion coefficient, the parameter ψ is the inverse of the Frisch elasticity, and ϕ is the scaling factor. The composite consumption is defined as

$$C_t = (a^{\frac{1}{\eta}} C_{H,t}{}^{\frac{\eta-1}{\eta}} + (1-a)^{\frac{1}{\eta}} C_{F,t}{}^{\frac{\eta-1}{\eta}})^{\frac{\eta}{\eta-1}} \tag{4}$$

where $C_{H,t}$ represents home goods and $C_{F,t}$ represents foreign goods. The elasticity of substitution between home and foreign goods is given by $\eta \in (0, +\infty)$ and $a \in (0,1)$ is the weight on home goods in the composite consumption index, which reflects the degree of home bias in preferences. The composite home good $C_{H,t}$ comprises a continuum of differentiated goods:

$$C_{H,t} \equiv \left(\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{5}$$

where $\varepsilon > 1$ represents the elasticity of substitution between any two differentiated home goods.

4.2. Budget constraints

A representative household maximizes its lifetime utility given by equation (1) subject to the following budget constraint:

$$P_t C_t + e_t B_t^* + \frac{\psi_B}{2} e_t B_t^{*2} + B_t = W_t N_t + R_{t-1}^* e_t B_{t-1}^* + R_{t-1} B_{t-1} + T_t \tag{6}$$

where B_t and B_t^* represent one-period risk-free nominal bonds denominated in domestic and foreign currencies, respectively. The nominal exchange rate is denoted by e_t and the gross nominal interest rates for the two types of bonds are denoted by R_t and R_t^*, respectively. W_t is the nominal wage and and N_t is the labor supply. T_t contains lump sum taxes and transfers from the government and profits from firms.

ψ_B, a quadratic portfolio holding cost for foreign bond holdings, serves two purposes in the model. When no capital control is imposed, the value of this parameter is negligible, only to ensure the stationarity of the system (Schmitt-Grohé and Uribe, 2003). Alternatively, the central bank can impose capital controls to smooth international capital flows, which appears as a wedge in the intertemporal Euler equation (Chang, Liu, and Spiegel, 2015).[3]

The total expenditure needed to attain a consumption index C_t is given by P_tC_t where P_t is defined as

$$P_t = \left[aP_{H,t}^{1-\eta} + (1-a)P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{7}$$

$P_{H,t}$, the price index of home goods, is defined as

$$P_{H,t} = \left[\int_0^1 P_{H,t}(j)^{1-\varepsilon}dj\right]^{\frac{1}{1-\varepsilon}} \tag{8}$$

4.3. Households' optimality condition

A set of intertemporal optimality conditions arises from the representative household's bond-holding decisions. The optimality condition derived from the holding of domestic currency denominated bond is given by:

$$E_t\left[\beta_t R_t\left(\frac{P_t}{P_{t+1}}\right)\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\right] = 1 \tag{9}$$

The following optimality condition applies to the holding of foreign currency-denominated bonds:

$$E_t\left[\beta_t\frac{R_t^*}{1+\psi_B B_t^*}\left(\frac{q_{t+1}}{q_t}\right)\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\right] = 1 \tag{10}$$

where ψ_B appears as a wedge in the Euler equation, as mentioned above. Without capital controls, the uncovered interest rate parity (UIP) holds in expectation. Thus, capital controls limit capital flows by reducing effective interest rate differentials.

Apart from consumption decisions, the representative household also decides how much

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[3] I assume that, in the baseline model, there is no capital control in place. The optimal capital control policy will be discussed as an extension. Notably, capital controls affect households' intertemporal decisions without altering their budget constraints; any tax revenues or subsidy costs will be transferred to households in a lump sum fashion.

labor to supply each period. The marginal utility of wage income equates to the marginal disutility of the labor supply.

$$\psi {C_t}^{\sigma} {N_t}^{\phi} = \frac{W_t}{P_t} \tag{11}$$

4.4. Production

Firms use a linear technology in labor, $Y_t(j) = A_t N_t(j)$, to produce home goods and they are subject to common productivity shock A_t, given by:

$$log(\frac{A_t}{\bar{A}}) = \rho_a log(\frac{A_{t-1}}{\bar{A}}) + \varepsilon_t^a, \quad \varepsilon_t^a \sim N(0, \sigma_a^2) \tag{12}$$

Following Calvo (1983), I assume that a fraction $\theta \in (0, 1)$ of firms cannot change their prices in each period. The remaining firms choose optimal reset prices to maximize their discounted future profits.

$$\max_{P_{H,t}(j)} E_t \sum_{s=0}^{\infty} \{(\theta)^s Q_{t,t+k} \left[P_{H,t}(j) - MC_{H,t+s}\right] Y_{H,t+s}(j)\} \tag{13}$$

where MC denotes the marginal cost of production in nominal terms and $Q_{t,t+k}$ is the stochastic discount factor.

4.5. External variables

Our small open economy is heavily influenced by the external environment through two major channels, foreign demand for home goods and the foreign interest rate. Aggregate demand for home goods is the sum of domestic and foreign demand, which is given by:

$$Y_{H,t} = a \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + (1-a) \left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta} C_t^* \tag{14}$$

where C_t^* is aggregate foreign demand, which is normalized such that in the steady state it is equal to the steady-state level of aggregate home consumption. I assume that aggregate foreign demand follows an AR(1) process given by:

$$log\left(\frac{C_t^*}{\bar{C}^*}\right) = \rho_c log\left(\frac{C_{t-1}^*}{\bar{C}^*}\right) + \varepsilon_t^c, \quad \varepsilon_t^c \sim N(0, \sigma_c^2) \tag{15}$$

More importantly, emerging market economies face recurrent episodes of persistent foreign interest rate shocks, leading to volatile capital flows. To capture this feature, I allow the foreign interest rate process to be regime-specific. Under the low-volatility regime, foreign interest rates follow an AR(1) process with a smaller persistence parameter and less variance in shocks. Under the high-volatility regime, the foreign interest rate process is more persistent and the shock variance is larger.

$$log\left(\frac{R_t^*}{\bar{R}^*}\right) = \rho_{r,s_t} log\left(\frac{R_t^*}{\bar{R}^*}\right) + \varepsilon_{t,s_t}^r, \quad \varepsilon_t^r \sim N(0, {\sigma_{s_t}^r}^2) \tag{16}$$

4.6. Regime switches

To capture the recurrent episodes of volatile capital flows in emerging market economies, I allow the economy to switch between two underlying regimes: the high-volatility regime (H) and the low-volatility regime (L). The regime switches according to a Markov chain and its transition matrix is denoted as Π, such that $\pi_{i,j}$ represents the probability of switching to the j^{th} regime from the i^{th} regime. The transition matrix Π is given by:

$$\Pi = \begin{pmatrix} p_{HH} & 1 - p_{HH} \\ 1 - p_{LL} & p_{LL} \end{pmatrix} \tag{17}$$

Consequently, the foreign interest rate process is characterized by persistence coefficient $\rho_{r,H}$ and shock standard deviation σ_H^r under the high-volatility regime, and $\rho_{r,L}$ and σ_L^r under the low-volatility regime.

More importantly, the central bank can choose whether to impose a regime-switching monetary policy or stick to a constant inflation-targeting rule. If the central bank operates differentially across regimes, its policy regimes can be characterized as regime-dependent response coefficients.

4.7. Monetary policy

Aggregate inflation is defined as $\pi_t = P_t/P_{t-1}$ and home goods inflation as $\pi_{H,t} = P_{H,t}/P_{H,t-1}$. The steady state is characterized by zero inflation.

The central bank determines the short-term nominal interest rate R_t according to a simple inflation-targeting rule, by which interest rates respond to inflation and possibly to exchange-rate fluctuations (see, e.g., Lubik and Schorfheide, 2007):

$$log\left(\frac{R_t}{\bar{R}}\right) = \rho\ log\left(\frac{R_{t-1}}{\bar{R}}\right) + (1-\rho)\left[\phi_\pi log\left(\frac{\pi_t}{\bar{\pi}}\right) + \phi_y log\left(\frac{Y_t}{\bar{Y}_t}\right) + \phi_e log\left(\frac{q_t}{q_{t-1}}\right)\right] \tag{18}$$

where $\bar{\pi}$, $\bar{Y}$, and $\bar{R}$ are the steady state values of inflation, output, and the nominal interest rate. The term ρ is the interest rate smoothing parameter. ϕ_π, ϕ_y and ϕ_e are the weights assigned by the central banker to the deviations of inflation and output from their steady-state levels and to fluctuations of the real exchange rate.[4] Setting the parameter ϕ_e at zero implies a pure inflation-targeting regime, under which the central bank responds only to inflation. In contrast, a positive ϕ_e indicates the will of stabilizing real exchange rate fluctuations.

When the coefficients in the monetary policy rule depend on the external volatility regime, I call it a regime-switching policy rule. The main goal of the paper is to study whether regime-switching monetary policy rules are more desirable given that emerging market economies face time-varying external volatilities.

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[4] This formulation is in line with the stated objective of many emerging market central banks, namely "leaning against the wind".

4.8. Solution methods

MS-DSGE models have received increasing attention recently as they incorporate the richness of regime switches into a DSGE model. Davig and Leeper (2007) develops a MS-DSGE model to study the interaction and regime switches of fiscal and monetary policy. Bianchi (2013) estimates a medium-scale MS-DSGE model with Bayesian techniques and finds repeated fluctuations of monetary policy between hawkish and dovish regimes. Farmer, Waggoner, and Zha (2011) and Foerster et al. (2014) develop powerful techniques to solve MS-DSGE models.

In this paper, I use the minimal state variables solution algorithm developed by Farmer, Waggoner, and Zha (2011) due to its robustness and speed of convergence. The outline of the algorithm and my approach to implementing it are given below:

First, I log-linearize the system around its steady state for each specific regime and write the system of equations in the following form:

$$A(s_t)x_t = B(s_t)x_{t-1} + \Psi(s_t)\varepsilon_t + \Pi(s_t)\eta_t, \tag{19}$$

where the parameter matrices depend on the regime state in period t. x_t is a vector of endogenous and predetermined variables, ε_t is a vector of exogenous shocks, and η_t is a vector of expectational errors.

Second, I implement the algorithm in Farmer, Waggoner, and Zha (2011) and transform the problem into one of finding the roots of quadratic polynomial functions. Then x_t and η_t can be written as the linear transformation of x_{t-1} and ε_t, given by:

$$x_t = V_{s_t}F_{1,s_t}x_{t-1} + V_{s_t}G_{1,s_t}\varepsilon_t \tag{20}$$

$$\eta_t = -(F_{2,s_t}x_{t-1} + G_{2,s_t}\varepsilon_t) \tag{21}$$

where $V_{s_t}, F_{1,s_t}, F_{2,s_t}, G_{1,s_t}$, and G_{2,s_t} are matrices solved based on the quadractic polynomial functions. Equations (20) and (21) are solutions to the system characterized by (19).

Finally, there can be multiple minimal state variables solutions for a given MS-DSGE model. To find all solutions, I randomly select from a number of initial values and compute the corresponding solutions until they converge. Then, as suggested by Farmer, Waggoner, and Zha (2011), there is a mapping from the root of the equation to the solution of the model. According to Proposition 3.9, p.36 and Proposition 3.33, p.49 in Costa, Fragoso, and Marques (2005), the candidate solution is stationary (mean-square-stable) if and only if all eigenvalues of the matrix in equation (10) of Farmer, Waggoner, and Zha (2011) are inside the unit circle.

As a result, checking the stationarity of the solution is equivalent to checking the size of the dominant eigenvalue of that matrix. If there is only one solution with dominant eigenvalue smaller than 1, there is no need for selection. If there are multiple solutions with dominant eigenvalues smaller than 1, one can use the likelihood method and select the equilibrium that delivers the highest likelihood value with respect to the data. This is especially appealing when estimating the model using real data.

4.9. Welfare function

A well-defined micro-founded welfare criterion is needed in order to compare policy choices. In general, there are two major ways to evaluate welfare: the linear-quadratic approach and the second-order approximation approach. The linear-quadratic approach obtains an analytic expression of the welfare function by approximating the equilibrium conditions up to the first order and the welfare function up to the second order Rotemberg and Woodford (1997). However, it may sometimes generate spurious results because some important second-order terms are ignored (Kim and Kim, 2003).

Alternatively, Schmitt-Grohé and Uribe (2004, 2007) develop a numerical method to evaluate welfare under various policy environments by approximating the whole system of equations up to the second order. While this method is accurate, the lack of an analytic expression of the welfare function prohibits further theoretical analysis. Furthermore, it does not apply to MS-DSGE models.

In this paper, I use the pure quadratic approximation method proposed by Benigno and Woodford (2012) to derive the welfare function analytically. To be more specific, I approximate the welfare function to the second order and eliminate all the first-order terms using equilibrium conditions. Unlike the traditional linear-quadratic method, the equilibrium conditions are approximated to the second order in order to get a pure quadratic approximation of the welfare function.

According to the derivation in Appendix B, the loss function based on the pure quadratic approximation can be written in the following fashion:

$$L_{t_0} = U_c \bar{C} E_{t_0} \sum \beta^t \left[\frac{1}{2} y_t' L_y y_t + y_t' L_e e_t + \frac{1}{2} l_{\hat{\pi}} (\hat{\pi}_t^H)^2 \right] + t.i.p. + \mathcal{O}(||\xi||^3) \tag{22}$$

where $y_t = [\hat{y}_t, \hat{c}_t, \hat{p}_{H,t}, \hat{Q}_t]$ are endogenous variables, $e_t = [\hat{a}_t, \hat{i}_t^*, \hat{c}_t^*]$ are external shocks, and L_t, L_e, and $l_{\hat{\pi}}$ are matrices and the scalar derived in the appendix.

Thus, not only do output and inflation enter the loss function, the real exchange rate also plays a significant role, which is consistent with De Paoli (2009). Based on the loss function, I compare alternative policy choices in the MS-DSGE model.

5. Data and Estimation

I estimate my DSGE model using data from Thailand. I use Thailand data because Thailand was one of the first emerging market economies to adopt inflation targeting so using data from Thailand gives me a longer time series. The sample ranges from 2000Q1 through 2015Q2 because Thailand adopted inflation targeting in 2000. A thorough guide to MS-DSGE model estimation can be found in Bianchi (2013).

5.1. Data

I estimate the model using five observable data series: inflation, the GDP growth rate, the nominal effective exchange rate, the real effective exchange rate, and the domestic interest rate. All data are retrieved from the CEIC database. I use X-13 filter to compute the seasonally adjusted quarter-by-quarter GDP growth rate. To equate the number of observable variables with the number of shocks, I add two additional types of shocks which are common in the literature, i.e., monetary policy shocks and foreign inflation shocks.

5.2. Prior values

Calibrating such a model is challenging as there is no consensus on the values of some parameters for emerging market economies. As a result, I pick parameter values from the existing literature as the means of the prior distributions and use Bayesian methods to estimate the model. Prior means and posterior modes are summarized in Table 2. The time period in the model is equivalent to one quarter.

I choose $\beta = 0.995$, which is equivalent to the observed annual real interest rate of 2 percent. The prior mean of σ, the risk-aversion parameter, is set at 2, a value commonly used in the literature on emerging market economies (Aguiar and Gopinath, 2007; Anand, Prasad, and Zhang, 2015; García-Cicco, Pancrazi, and Uribe, 2010). The share of home-produced tradable goods, denoted by a, is set at 0.7, which implies that home goods account for 70% of domestic consumption (Galí and Monacelli, 2005).

The elasticity of substitution between home and foreign goods is assumed to be 1 (Obstfeld and Rogoff, 2005). The Frisch elasticity ($1/\psi$) is assumed to be 1/3 (in other words, $\psi = 3$). For the monetary policy parameters, I use loose priors and choose $\rho = 0.5$, $\phi_\pi = 1.5$, $\phi_e = 0.25$, $\phi_y = 0.125$.

Lastly, I use the estimates of the foreign interest rate processes to pin down the parameter values that are regime-specific. For the persistence parameter of the foreign interest rate

Table 2. Bayesian Estimation Results

Parameter	Prior		Posterior Mode	
	mean	*std. dev.*	*00Q1 - 07Q2*	*07Q3 - 15Q2*
β	0.995	0.005	0.996	0.996
σ	2	0.75	2.57	2.24
a	0.7	0.1	0.81	0.78
η	1	0.5	0.87	1.01
ψ	3	1	2.53	2.33
ρ	0.5	0.2	0.92	0.94
ϕ_π	1.5	0.5	1.93	1.28
ϕ_e	0.25	0.13	0.16	0.28
ϕ_y	0.125	0.07	0.08	0.09

Notes: Parameters are estimated using the Random-walk Metropolis algorithm with 100,000 draws. The prior distributions are borrowed from the existing literature.

process, ρ_r, I set the value at 0.93 based on the median of the estimates. The standard deviations of shocks are set at (0.1) under the high- (low-) volatility regime.

5.3. Bayesian estimation

Based on the above-mentioned prior values, I implement Bayesian estimation using Thailand macroeconomic data from 2010Q1 - 2015Q2. I split the sample into two sub-samples, the pre-financial crisis period and the post-financial crisis period. The results are summarized in Table 2.

The estimates of structural variables are very close under both regimes while the interest rate rule parameters and shock parameters are very different, consistent with the regime-switching assumption. Based on the parameter values from Bayesian estimation results, I then compute the impulse responses and welfare outcomes under various policy choices.

6. Main Results

Based upon the model developed and estimated above, I now examine three aspects of the desirability of regime-switching policy rules. First, it is crucial to verify that regime-switching policy rules ensure macroeconomic stability. Otherwise, they are strictly dominated by any inflation-targeting rules that can achieve this goal. This can be done by examining the existence and uniqueness of rational-expectation equilibrium under a wide range of policy coefficients.

Second, it is useful to know whether regime-switching monetary policy rules would lead to more stable aggregate output when the home economy faces large foreign interest rate shocks. The major reason that emerging market central banks deviate temporarily from inflation-targeting rules and stabilize their exchange rates is that targeting inflation is not sufficient to stabilize the home economy. This can be analyzed by comparing impulse responses to foreign interest rate shocks under inflation-targeting rules and regime-switching rules.

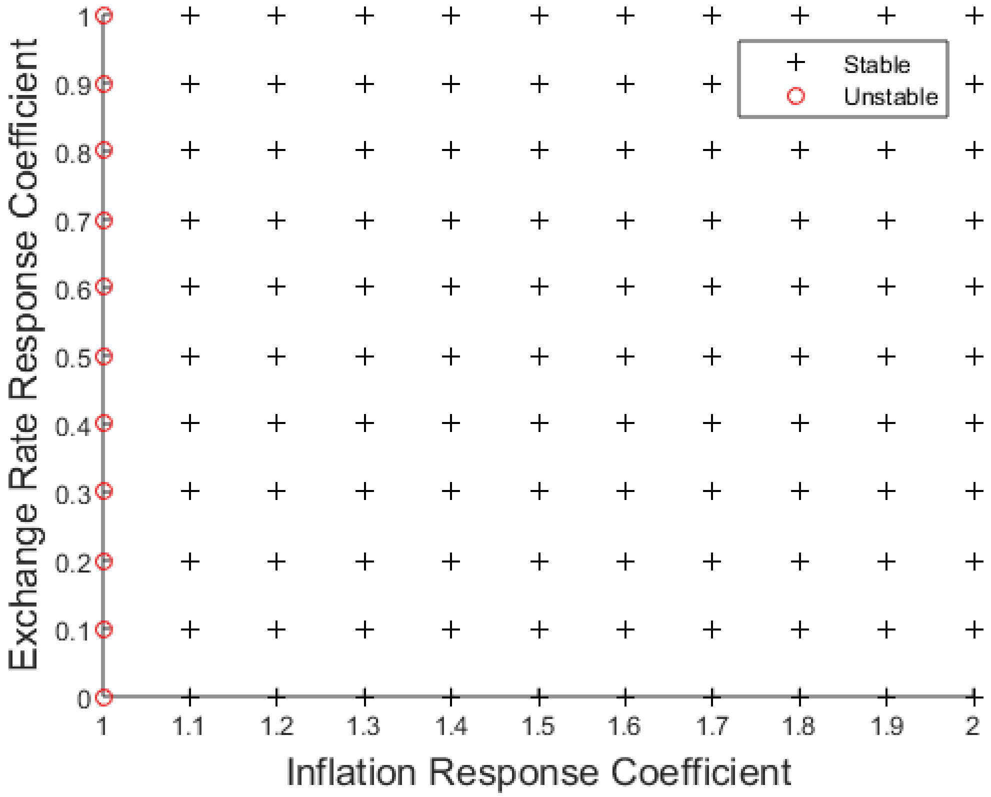
Lastly, it is important to understand the welfare outcomes when regime-switching policy rules are allowed. Using the method discussed above, I can approximate the welfare function up to the second order and numerically calculate the welfare levels under various types of policy rules. This allows me to study the welfare outcomes and search for the optimal simple rule.

6.1. Macroeconomic stability

A major advantage of operating monetary policy based on explicit interest rate rules is that it ensures macroeconomic stability. For example, the famous Taylor principle suggests that in order to ensure macroeconomic stability, the central bank should adjust interest rates more than one-to-one in response to changes in inflation rates (Davig and Leeper, 2007).

In practice, ensuring macroeconomic stability is not an easy task, as some seemingly reasonable monetary policy rules will in fact cause macroeconomic instability. For example,

Figure 2. Macroeconomic Stability under Varying Policy Coefficients



Notes: This figure shows the stability status of the economy conditional on varying values of inflation and exchange-rate response coefficients. A black plus sign means that the rational-expectation equilibrium uniquely exists while a red circle implies that the equilibrium is either non-unique or does not exist. The interest rate smoothing coefficient ρ is set at 0.7 and the inflation response coefficient ϕ_π at 2, which are plausible parameter values according to the estimation.

Uribe (2003) finds that targeting the real exchange rate can generate aggregate instability due to self-fulfilling expectations.

Using the model developed and estimated above, I examine the existence and uniqueness of the rational-expectation equilibrium under a wide range of policy coefficients. Generally speaking, macroeconomic stability is ensured under such regime-switching policy rules. A detailed example is given in Figure 2, in which I present the stability status under varying values of ϕ_π and ϕ_e, which are coefficients governing the interest rate response to inflation and exchange-rate fluctuations.

According to Figure 2, as long as the Taylor principle is met, namely, $\phi_\pi > 1$, macroeconomic stability is achieved. Thus, for an emerging market central bank, adopting regime-switching monetary policy rules is not going to impose any additional limits compared with pure inflation-targeting rules. This is a very desirable property.

6.2. Impulse responses

Since emerging market central banks are concerned with large economic fluctuations that are due to foreign interest rate shocks, I now examine whether temporarily smoothing exchange rates can effectively stabilize the domestic economy. To be more specific, I compare impulse response functions under the pure inflation-targeting rule and a regime-switching rule that temporarily smooths exchange rates. The home economy is under the high-volatility regime and faces a one-standard-deviation positive foreign interest rate shock, which can be thought of as a sudden stop. The impulse responses are plotted in Figure 3.

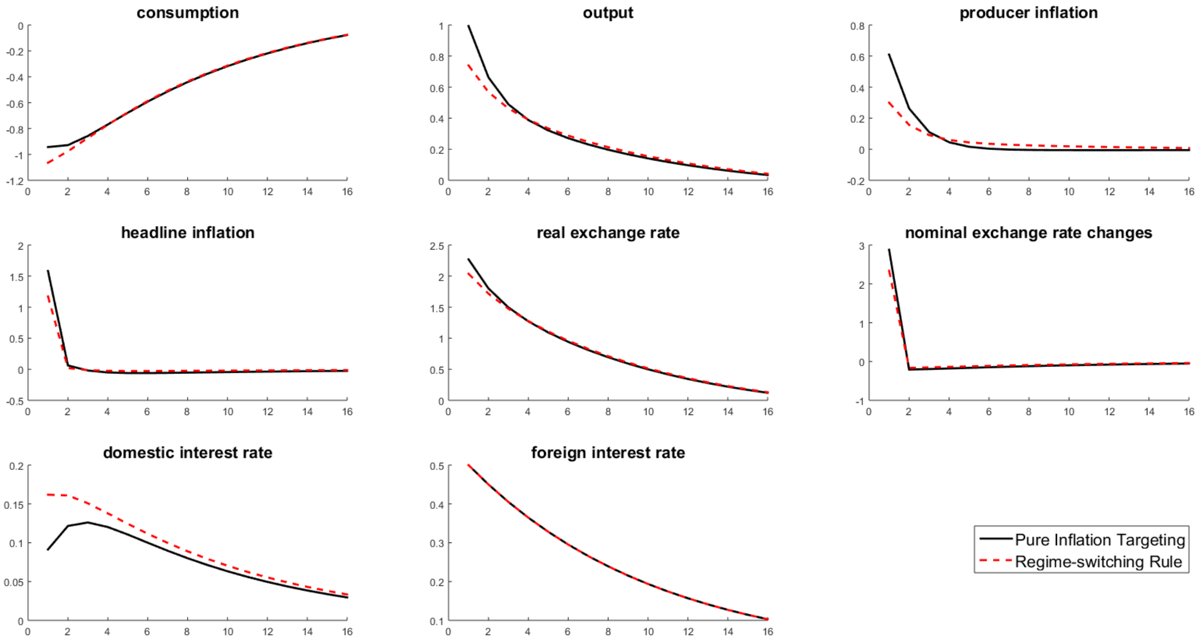
The impulse responses show that temporary exchange rate smoothing leads to a higher domestic interest rate, which smooths nominal and real exchange rates as well as domestic inflation and output. A higher domestic interest rate reduces currency depreciation and capital outflows. It also stabilizes output by reducing producer inflation. Thus, monetary policy that temporarily smooths exchange rates can help stabilize aggregate output in the short run.

6.3. Welfare analysis of regime-switching monetary policy

While regime-switching monetary policy that temporarily smooths exchange rates can mitigate the fluctuation of aggregate output during periods of volatile capital flows, its desirability remains questionable unless its welfare properties are well understood.

Conceptually, regime-switching monetary policy that temporarily smooths exchange rates should be welfare-improving compared with pure inflation-targeting rules because large foreign interest rate shocks cause volatile capital flows and large fluctuations in net exports. As a result, the home economy incurs welfare loss because of inefficient fluctuations in the

Figure 3. Impulse Responses to a Positive Foreign Interest Rate Shock (High-volatility Regime)



Notes: This figure shows the responses of several variables to a one-standard-deviation foreign interest rate shock under the high-volatility regime. Two policy rules are considered. The black solid lines are impulse responses under the pure inflation-targeting rule and the red dashed lines represent impulse responses under the regime-switching rule (temporary exchange rate smoothing). The responses are all expressed as percentage deviations from the steady-state values of the corresponding variables. An increase in the exchange rate (both nominal and real) indicates depreciation.

terms of trade due to nominal rigidities. By temporarily smoothing exchange rates, emerging market central banks can smooth production and consequently stabilize the terms of trade.

Using the method explained above, I approximate the welfare function up to the second order and compute welfare outcomes under various monetary policy rules. In particular, I look for the optimal simple rule by searching for the coefficient combination that maximizes welfare.[5]

There is one particularly interesting feature of the pattern of optimal monetary policy. If

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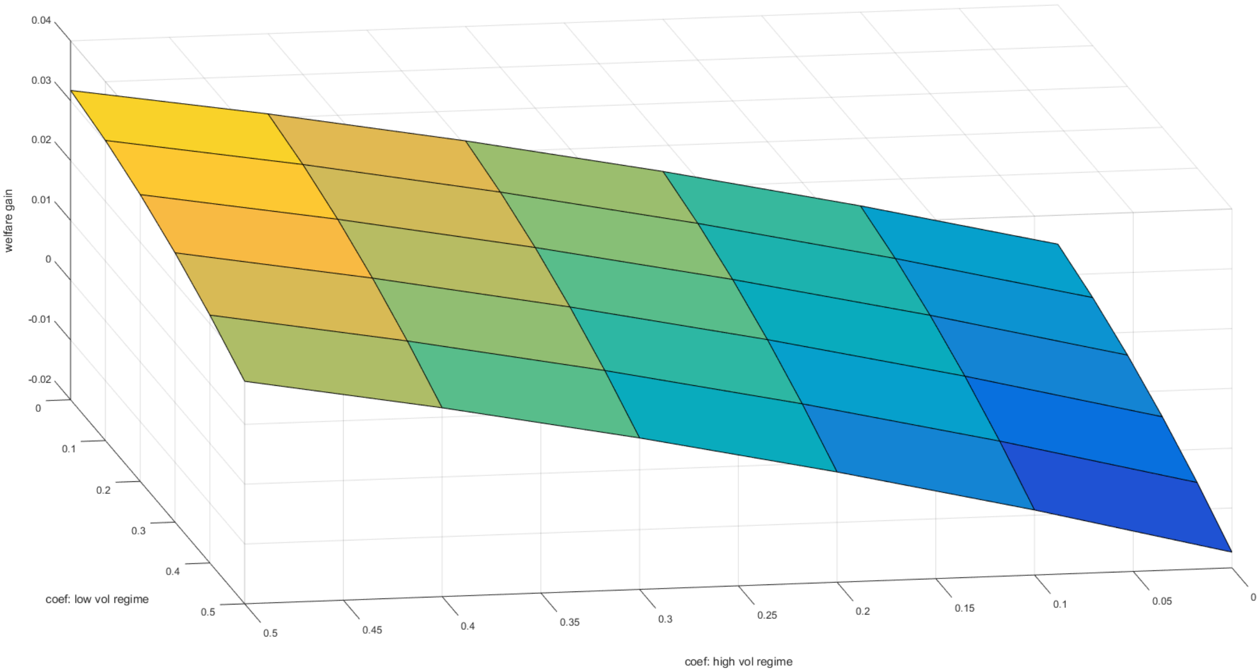
[5] Detailed results are available in the online appendix. I use constrained optimization methods to search for the optimal simple rule.

Figure 4. Welfare Gains under Various Regime-switching Policy Coefficients



Notes: This figure shows the relative welfare gains under various values of ϕ_e. The interest rate smoothing coefficient ρ is set at 0.7, the inflation response coefficient ϕ_π is set at 2, and the output gap response coefficient ρ_y is set at 0, which are plausible parameter values according to the estimation. The two axes on the plane correspond to the exchange rate response coefficients under the high-volatility regime and the low-volatility regime. The vertical axis displays the relative welfare gain. The result suggests that welfare is higher if ϕ_e is zero under the low-volatility regime and very high under the high-volatility regime. The results are robust under alternative parameter values.

I control for the other coefficients and vary the response to exchange-rate fluctuations under contrasting volatility regimes, it is clear that the welfare level is higher when there is no response to exchange-rate fluctuations under the low-volatility regime and strong response under the high-volatility regime. This relationship is shown in Figure 4.

In fact, the optimal simple rule contains some “leaning against the wind” features as it suggests that the central bank should respond to exchange-rate fluctuations only under the high-volatility regime. This is because, under the low volatility regime, the existence of productivity shocks and foreign demand shocks creates trade-offs between stabilizing inflation and the exchange rate, so the optimal operating rule should not respond to the exchange rate. Under the high-volatility regime, however, foreign interest rate shocks become the most important source of disturbance so monetary policy should respond to exchange-rate fluctuations aggressively.

6.4. Capital controls

In the baseline model, I assume that no capital controls are imposed and households can freely borrow and save via the international financial market. However, many emerging market central banks impose temporary capital controls when facing volatile capital flows and it is important to understand the consequences of such intervention.

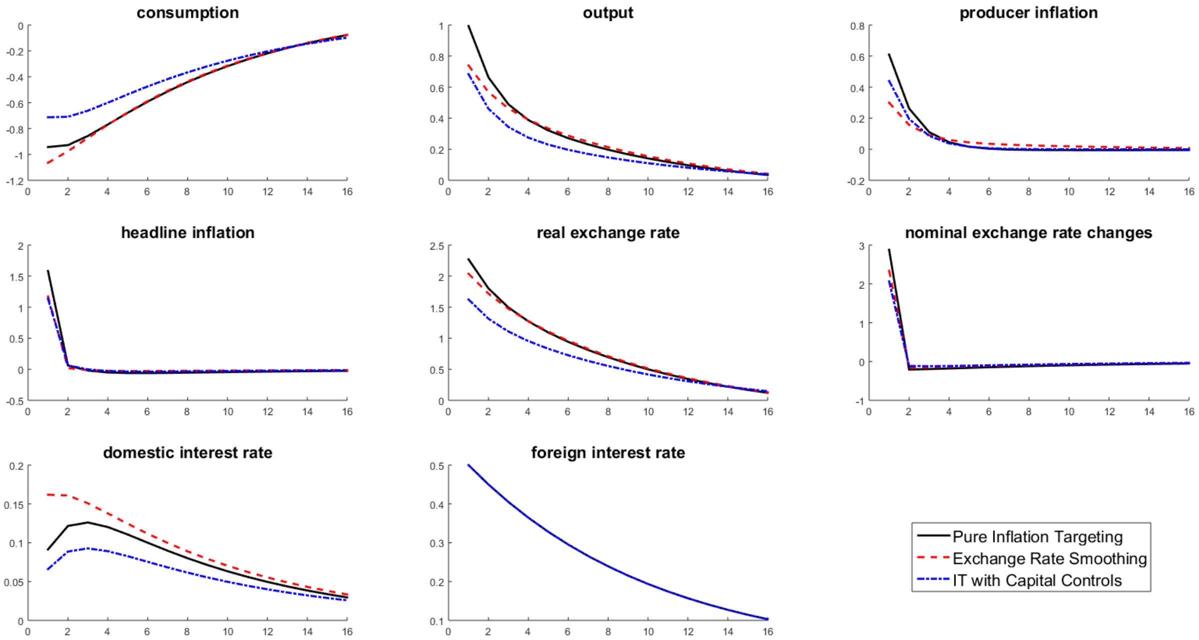
I model capital controls as quadratic portfolio adjustment costs when households change their bond holdings $\Gamma_t(B^*_t, B^*_{t-1}) = \psi_B(B^*_t - B^*_{t-1})^2$. Capital controls reduce capital flow volatility by inserting a wedge in the intertemporal Euler equation, as shown below. Notably, any costs or revenues from capital controls are transferred to households in lump sum fashion so that capital controls have no wealth effect.

$$E_t \left\{ \beta_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left[\frac{R^*_t}{1 + \psi_B(B^*_t - B^*_{t-1})} \frac{q_{t+1}}{q_t} - \frac{R_t}{\pi_{t+1}} \right] \right\} = 0 \tag{23}$$

I now use the extended model to study the implications of imposing temporary capital controls. First, temporary capital controls will not jeopardize macroeconomic stability. Furthermore, imposing temporary capital controls can effectively stabilize aggregate output and consumption when the home economy faces large foreign interest rate shocks. This can be found in the impulse responses shown in Figure 5.

According to Figure 5, capital controls are much more effective at stabilizing aggregate output and consumption because they substantially reduce capital flows in the short run. The wedge placed in the intertemporal Euler equation gives the central bank a certain degree of flexibility as it does not need to adjust the domestic interest rate to minimize capital flows. With the adjustment cost, households are no longer as willing to save or borrow in foreign

Figure 5. Impulse Responses to a Positive Foreign Interest Rate Shock (with Capital Controls)



Notes: This figure shows the responses of several variables to a one-standard-deviation foreign interest rate shock under the high-volatility regime. Three policy rules are considered. The black solid lines are impulse responses under a pure inflation-targeting rule, the red dashed lines represent impulse responses under a temporary exchange-rate smoothing rule, and the blue dash-dot lines show impulse responses under an inflation-targeting rule with temporary capital controls. The responses are all expressed as percentage deviations from the steady-state values of the corresponding variables. An increase in the exchange rate (both nominal and real) indicates depreciation.

currency denominated bonds, so aggregate output and consumption are less volatile given temporary capital controls.

I then study the welfare effect of capital controls by allowing the central bank to incorporate capital controls as part of the regime-switching policy rules and impose them only under the high-volatility regime. It turns out that capital controls are welfare-reducing because households, anticipating possible capital controls in the future, will behave inefficiently under such circumstances. Detailed welfare comparisons are available in the online appendix.

7. Conclusion

Emerging market economies frequently experience volatile capital flows and considerable exchange-rate fluctuations caused by large foreign interest rate shocks, so it is natural to ask how central banks should react when external volatility is high. Using data from emerging market economies, I develop a Markov-switching dynamic stochastic general equilibrium (MS-DSGE) model that features external shocks of time-varying volatilities to study their monetary policy choices.

I find that by allowing the response coefficients in the interest rate rule to vary according to the regime of external volatility, the central bank can improve welfare while maintaining macroeconomic stability. The optimal simple rule suggests that the central bank should target inflation when external volatility is low and stabilize exchange rates when it is high, which is akin to the "leaning against the wind" approach adopted by many emerging market central banks. Temporary capital controls are effective in terms of short-term stabilization but are inferior in terms of welfare outcomes.

One interesting direction for future research would be relaxing the assumption of perfect information between households and the central bank. The first scenario would be that the state of external volatility is observable but households do not have full information of the monetary policy rule under this regime, so they have to learn from the observed policy rates. This will no doubt reduce the effectiveness of monetary policy and it is a very realistic scenario in emerging market economies given the weak reputation of the central banks.

The second scenario would be that the state of external volatility is unobservable but the policy regime is, so households learn the state of the economy from the central bank policy regime. If so, policy regime switches can be potentially detrimental because central banks may create and amplify economic crises by making the state of the economy explicit. This is also possible given information frictions in emerging market economies.

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Appendix A: The Competitive Equilibrium of the MS-DSGE Model

A.1 Profit-maximization problem

Here, the profit-maximization problem is similar to that presented in Galí and Monacelli (2005). Firms maximize profits given in equation (12) and the optimality condition is given by:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y_{t+k} \left(P_{H,t}^* - \frac{\varepsilon}{\varepsilon - 1} MC_{t+k} \right) \right\} \tag{24}$$

where MC is the nominal marginal cost. Following the traditional literature, I introduce an employment subsidy τ such that $1 - \tau = \frac{\varepsilon}{\varepsilon - 1}$ so the flexible price equilibrium is efficient and the goal of the monetary policy is to eliminate inefficiency due to nominal rigidities.

Then, I log-linearize the above first-order condition around its steady state:

$$p_{H,t}^* = p_{H,t-1} + \sum_{k=0}^{\infty} (\beta\theta)^k E_t\{\pi_{H,t+k}\} + (1 - \beta\theta)^k E_t\{\widehat{mc}_{t+k}\} \tag{25}$$

Combining it with the price distribution, the inflation dynamics of home goods can be written as below:

$$\widehat{\pi}_{H,t} = \beta E_t\{\widehat{\pi}_{H,t-1}\} + \frac{(1-\theta)(1-\beta\theta)}{\theta} \widehat{mc}_t \tag{26}$$

Lastly, the deviation of real marginal cost denoted by home goods can be written as below:

$$\begin{aligned} \widehat{mc}_t &= \sigma \widehat{c}_t + \phi \widehat{n}_t + \tfrac{1-a}{a} \widehat{q}_t - \widehat{a}_t \\ &= \sigma \widehat{c}_t + \phi \widehat{y}_t + \tfrac{1-a}{a} \widehat{q}_t - (1+\phi) \widehat{a}_t \end{aligned} \tag{27}$$

where $\widehat{q}_t$ is the percentage deviation of the real exchange rate from its steady state.

To sump up, the aggregate supply function can be written as:

$$\widehat{\pi}_{H,t} = \beta E_t\{\widehat{\pi}_{H,t-1}\} + \frac{(1-\theta)(1-\beta\theta)}{\theta} \left(\sigma \widehat{c}_t + \phi \widehat{y}_t + \frac{1-a}{a} \widehat{q}_t - (1+\phi) \widehat{a}_t \right) \tag{28}$$

A.2 Demand for home goods

Demand for home goods is the sum of domestic demand and foreign demand. By the market clearing condition, total demand is equal to total supply. Domestic demand for home goods is $a \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t$ from the household optimization problem. Aggregate foreign demand C_t^* is normalized such that its steady-state value is equal to the steady-state level of domestic output. Assuming a similar preference structure, foreign demand for home goods is $(1-a)\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta} C_t^*$. Therefore, demand for home goods can be written as equation (14). Furthermore, the log-linearized relationship can be written as

$$\begin{aligned} \widehat{y}_t &= a[-\eta(-\tfrac{1-a}{a})\widehat{q}_t + \widehat{c}_t] + (1-a)[-\eta(-\tfrac{1}{a})\widehat{q}_t + \widehat{c}_t^*] \\ &= \tfrac{1-a^2}{a}\eta\widehat{q}_t + a\widehat{c}_t + (1-a)\widehat{c}_t^* \end{aligned} \tag{29}$$

A.3 The log-linearized system of equations

Competitive equilibrium can be characterized by a system of 11 equations and 11 variables, out of which 8 are endogenous and 3 are exogenous. Here I write out the complete log-linearized system of equations with variables $\{\widehat{c}_t, \widehat{\pi}_{H,t}, \widehat{q}_t, \widehat{\pi}_t, \widehat{y}_t, \widehat{b}_t^*, \widehat{\Delta e}_t, \widehat{a}_t, \widehat{i}_t, \widehat{i}_t^*, \widehat{c}_t^*\}$ as their percentage deviations from their steady state values.

The resource constraint of the small open economy:

$$\widehat{y}_t = \widehat{c}_t + \widehat{b}_t^* - \frac{1}{\beta}\widehat{b}_t^* \tag{30}$$

The log-linearized Euler equation from the domestic bond-holding decision:

$$\widehat{c}_t = E_t\widehat{c}_{t+1} - \frac{1}{\sigma}(\widehat{i}_t - E_t\widehat{\pi}_{t+1}) \tag{31}$$

The interest rate parity:

$$\widehat{i}_t - E_t\widehat{\pi}_{t+1} = \widehat{i}_t^* + E_t\widehat{q}_{t+1} - \widehat{q}_t - \psi_B\widehat{b}_t^* \tag{32}$$

The definition of inflation:

$$\widehat{\pi}_t = \widehat{\pi}_{H,t} + \frac{1-a}{a}(\widehat{q}_t - \widehat{q}_{t-1}) \tag{33}$$

The definition of the real exchange rate:

$$\widehat{q}_t - \widehat{q}_{t-1} = \widehat{\Delta e}_t - \widehat{\pi}_t \tag{34}$$

The interest rate rule:

$$\widehat{i}_t = \rho\widehat{i}_{t-1} + (1-\rho)(\phi_\pi\widehat{\pi}_t + \phi_e\widehat{\Delta e}_t) \tag{35}$$

The productivity process:

$$\widehat{a}_t = \rho_a\widehat{a}_{t-1} + \varepsilon_t^a \tag{36}$$

The foreign demand process:

$$\widehat{c}_t^* = \rho_c \widehat{c}_{t-1}^* + \varepsilon_t^c \tag{37}$$

The regime-switching foreign interest rate process:

$$\widehat{i}_t^* = \rho_r(s) \widehat{i}_{t-1}^* + \varepsilon_t^r(s) \tag{38}$$

Equations (28) - (38) constitute the competitive equilibrium of the MS-DSGE model and its solution can be found using the algorithm illustrated by Farmer, Waggoner, and Zha (2011).

Appendix B: Loss Function Derivation

I shall follow the notation in De Paoli (2009) for welfare function derivation. As mentioned above, the pure quadratic approximation is conducted by approximating both the welfare function and the relevant first-order conditions up to the second order so that I can cancel out all first-order terms.

To begin with, the second-order approximation of the welfare function is given below:

$$\begin{aligned} W_{t_0} &= U_c \bar{C} E_{t_0} \sum \beta^t \left[\hat{c}_t - \hat{y}_t + \tfrac{1-\sigma}{2} \hat{c}_t^2 - \tfrac{\phi+1}{2} (\hat{y}_t - \hat{a}_t)^2 \right] + t.i.p. + \mathcal{O}(||\xi||^3) \\ &= U_c \bar{C} E_{t_0} \sum \beta^t \left[w_y' y_t - \tfrac{1}{2} y_t' W_y y_t - y_t' W_e e_t - \tfrac{1}{2} w_{\hat{\pi}} \hat{\pi}_t^2 \right] \end{aligned} \tag{39}$$

where

$$w_{\hat{\pi}} = \frac{\varepsilon}{\kappa}, \quad w_y' = [-1, 1, 0]$$

$$W_y' = \begin{bmatrix} 1+\phi & 0 & 0 & 0 \\ 0 & -(1-\sigma) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, W_e' = \begin{bmatrix} -(1+\phi) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Then I need to approximate relevant first-order conditions up to the second order, including the real exchange rate definition, the intertemporal Euler equation, demand for home goods, and aggregate supply.

First, the vectors and matrices derived from the real exchange rate definition are:

$$f'y = [0, 0, -a, -(1-a)], \quad f'_e = [0, 0, 0]$$

$$F'_y = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -a(1-\eta) & 0 \\ 0 & 0 & 0 & -(1-a)(1-eta) \end{bmatrix}, F'_e = 0$$

Second, the vectors and matrices derived from the intertemporal Euler equation are:

$$c'y = [0, (1-\beta)\sigma, 0, (\beta-1)], \quad c'_e = [0, -\beta, 0]$$

$$C'_y = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & (\beta-1)\sigma^2 & 0 & \sigma(1-\beta) \\ 0 & 0 & 0 & 0 \\ 0 & \sigma(1-\beta) & 0 & (\beta-1) \end{bmatrix}, C'_e = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma\beta & 0 \\ 0 & 0 & 0 \\ 0 & \beta & 0 \end{bmatrix}$$

Third, the vectors and matrices derived from demand for home goods are:

$$d'y = [-1, a, -\eta, (1-a)\eta], \quad d'_e = [0, 0, 1]$$

$$D'_y = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & a(1-a) & 0 & -\eta a(1-a) \\ 0 & 0 & 0 & 0 \\ 0 & -\eta a(1-a) & 0 & \eta^2 a(1-a) \end{bmatrix}, D'_e = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -a(1-a) \\ 0 & 0 & 0 \\ 0 & 0 & \eta a(1-a) \end{bmatrix}$$

Finally, the vectors and matrices derived from aggregate supply are:

$$a'_y = [\phi, \sigma, -1, 0], \quad a'_\pi = (\phi+1)\frac{\sigma}{\kappa}$$

$$A'_y = \begin{bmatrix} \phi(2+\phi) & \sigma & -1 & 0 \\ \sigma & -\sigma^2 & \sigma & 0 \\ -1 & \sigma & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, A'_e = \begin{bmatrix} -\phi(1+\phi) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

After substituting the linear terms in the welfare function, I have:

$$[a_y \, d_y \, f_y \, c_y]L_x = w_y \tag{40}$$

The loss function can then be written as:

$$L_{t_0} = U_c\bar{C}E_{t_0}\sum\beta^t\left[\frac{1}{2}y_t'L_y y_t + y_t'L_e e_t + \frac{1}{2}l_{\hat{\pi}}(\hat{\pi}_t^H)^2\right] + t.i.p. + \mathcal{O}(||\xi||^3) \tag{41}$$

where

$$\begin{aligned} L_y &= W_y + L_x(1)A_y + L_x(2)D_y + L_x(3)F_y + L_x(4)C_y & (42)\\ L_e &= W_e + L_x(1)A_e + L_x(2)D_e + L_x(4)C_e & (43)\\ l_{\hat{\pi}} &= w_{\hat{\pi}} + L_x(1)a_{\hat{\pi}} & (44) \end{aligned}$$

The loss function is then used to compare welfare outcomes under a range of policy rules.