A Demographic Theory of Economic Reform¹

Y. Stephen Chiu²

University of Hong Kong

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²School of Economics and Finance, Faculty of Business and Economics, University of Hong Kong, Hong Kong. email: stephen.chiu@hku.hk

Abstract

This paper provides a new perspective to understanding the choices of reform strategies of China and Russia at the beginning of their reforms. China started its reform with a gradual approach that allows trials and errors, whereas Russia started its reform with a big bang approach. Since the gradualist reform approach is usually praised as one key factor for the success of the Chinese reform, it is important to delineate conditions under which the approach is viable. This paper presents a theoretic model that explores the role played by demography—age structure at the onset of reform and its dynamic—in addressing the issue. "The great events of history are often due to secular changes in the growth of population and other fundamental economic causes, which, escaping by their gradual character the notice of contemporary observers, are attributed to the follies of statesmen or the fanaticism of atheists." (J. M. Keynes, 1922, p.12)

"only a crisis—actual or perceived—produces real change. When that crisis occurs, the actions that are taken depend on the ideas that are lying around." (M. Friedman, in Friedman and Friedman, 1982, preface p. ix))

1 Introduction

This paper provides a new perspective to understanding the choices of reform strategies of China and Russia at the beginning of their reforms. China started its reform in 1978 with a gradual approach that allows steps to be reversed and experimentation. Russia, on the other hand, started its reform with a big bang approach in early 1990's. While the gradual reform approach is generally viewed as one main feature of the Chinese reform, the lesson is of limited value if we are uncertain about the circumstances under which the approach is applicable. In this paper, we explore the role of demography—both the structure and dynamic—in enabling gradual reform.

There are two reasons why we think the role of demography deserves examining. First, there were indeed stark differences between the two countries in terms of their demography. China's population was much younger than its Russian counterpart at the beginning of their reforms. Despite a small 1.1 years difference in 1950 (23.9 for China versus 25.0 for Russia), the difference in their median ages was widened to 9.2 years in 1980 (22.1 for China versus 31.2 for Russia). If we take 1980 and 1990 as the comparison years for China and Russia, respectively, then the difference was further widened to 11.2 years (Russia's median age was 33.2 in 1990). A difference in age structure has implications in the political economy of reform, when reform has different impacts on different generations of the population.

The second reason is that gradualism in China would be difficult to explain without taking its demography into account, and the same gradualism does not seem to have been possible in Russia because of its demography. It is useful to point to two important early works on China's reform. Qian and Xu (1993) argue that the distinguishing characteristic of Chinese gradual reform is its "sustained entry and expansion of the non-state sector,"¹ going on analyzing institutional details unique to China that enabled such phenomenon to occur. At the same time, Naughton (1995, 2007) examined the "growing-out-of-plan" reform strategy in the state sector: "[T]hroughout the 15 years of economic reform, between 1978 and about 1993, although the state sector had shrunk in relative importance, it had continued to *grow in absolute terms*, both in output *and in employment* (my emphasis)" (Naughton, 2007). This strategy allowed state enterprises to gradually adopt to market competition and operate as for-profit firms, not to mention to provide political stability.

Prescient as each analysis is, the concurrence of both features of reform hinges on a peculiar pre-condition of demography. China's population is not expected to peak until around 2030—five decades after the beginning of reform and despite its birth control policy—and by that time its size will be more than 40 percent larger than its 1980 population number.² Thus, understandingly, in a country like China, it is possible to have gradual expansion of the private sector without a contraction of its public sector, at least in the initial period, say the first one or two decades, of its reform. For a country like Russia, giving its shrinking population (its population peaked in 1993), the same would not be feasible, however. If the employment in the state sector were to be maintained, there would not be spare labor supply to the non-state sector; to ensure sustained expansion of the latter sector, drastic privatization and downsizing of the state sector would be inevitable. In other words, what has been described in Qian and Xu and Naughton could not have been feasible in Russia, simply because of its demography.

In this paper, we develop a theory to show how certain initial population characteristics – young population structure and reduced fertility rate — bestow a government with a

¹ "What makes China's reforms differ from those of Eastern Europe and the Soviet Union is the sustained entry and expansion of the non-state sector.....Our analysis have demonstrated that the success of China's particular gradual strategies depends on its initial institutional conditions (as well as other micro-and macroeconomic environment which are not discussed here),..."—Qian and Xu (1993; p.1 and p.44)

²The population size in 1980 was 977.8M (World Population Prospects: The 2015 Revision. United Nation, Department of Economic and Social Affairs, Population Division (2015)). According to the United Nations, the population size of China will peak in 2030 with the number of 1,416M (Population Estimates and Projections, World Bank Group. 01-July-2015).

large leeway in maneuvering so that economic reforms are less likely to be under popular resistance. The argument is twofold. The first point is related to population structure. Despite the long run benefits, even Pareto-improving reform may still be painful in the short run. While young people will live long enough to benefit from reform and are more likely to support it, it may not be the case for older people. It follows that a country with a young population structure like China is more likely to support the reform.

The second, more subtle point is the role played by the number of children to be had in a family. Notice that, despite the aforementioned point, even the young do not necessarily support reform when the hardship is too extreme. We argue that, in this case, the fewer children that young adults have or are expected to have may sway them into reform supporters. The reason is as follows. During a person's adulthood, the child-raising period occurs in his or her early stage and hence the burden of child raising is front loaded; the person's income profile, on the contrary, is more likely to be back loaded and is more so under reform than under no-reform because of the investment nature of the reform.³ Hence, inter-temporal consumption smoothing is more difficult to achieve under reform, and the extra hardship due to an increase in the number of children is felt more severely under reform than under no-reform. It follows that the smaller the number of children, the more tolerable the person is to reform hardship. This means that, in the case of China, the low fertility rate since the 1980s, partly due to the one-child policy, might have played a secret role in providing public support to its reform.

In an infinite-horizon overlapping-generation model in which agents live for four stages: children, young workers, middle-age workers, and retirees, we establish a general result: given different specifications about what will happen in the future should the present reform be delayed, the greatest hardship under reform that young workers can bear is decreasing in the number of children. This result is obtained under the assumption of CRRA utility function and for the range of value for the risk averse coefficient consistent to that is found in the quantitative macro literature; for other values, an opposite result is obtained.

³Under the reform scenario, the young people will undergo hardship in the short run but gain in the long run; under the no-reform scenario, they will not undergo hardship in the short run and will not gain the long run either. This suggests, in the case of indifference between the two options, the income profile must be more back loaded under reform than under no-reform.

Our theory has implications on the choice of reform strategies. Suppose there are two reform strategies. A gradual reform allows trials and errors and is more efficient, but it may be derailed or reversed under popular dissatisfaction, a big bang is not reversible once adopted, but it is less efficient. Now imagine what a benevolent government will do. If the country has favorable demographic conditions like China, then the government will be less constrained and will choose gradual reform thanks to its greater efficiency. Otherwise, if possible, the government will choose big bang to avoid future opposition that would occur if gradual reform is chosen.

A few remarks are in order. First, given the difficulty of moving from planned economy to market economy, the ideal reform package should be a gradual one. We assume the gradual reform considered in our model is indeed that ideal reform package, which in particular has taken into account the bundling of complementary components and nonetheless is presented in our model as a single policy choice.

Second, research has found that gradualism—in the form of divide-and-conquer strategy may break political resistance that may otherwise thwart big bang reform (Dewatripont and Roland 1992, 1993; Wei 1997). Their emphasis is that political constraints dictate that reform that should be ideally implemented in one go is feasible only when it is carefully sequenced in. Our emphasis, on the other hand, is that political constraints dictate that reform that should be ideally implemented sequentially is feasible only when it is implemented in one go. What is common between these two lines of thoughts is that the consideration of political constraints may lead to a compromise in the speed and package of policy choice.

Third, we consider a key merit of big bang in our framework is non-reversibility. Our emphasis of big bang reform as a commitment device is similar to the elite's extension of the franchise to the poor so as to commit to future re-distribution policy in Acemoglu and Robinson (2000). The importance of commitment over discretion in policy making is first famously made by Kyland and Prescott (1976). The commitment role of big bang has been emphasized by Murphy, Shleifer, and Vishny (1992) in that big bang could avert a partial reform trap that would otherwise result from gradual reform,

Fourth, the reader may wonder why, when not allowing a gradual reform to occur, de-

mography does not exert the same constraint on big bang reform so as to render it infeasible. One plausible reason is that a "window of opportunity" was present in Russia in early 1990s' when the government had a free hand in adopting drastic reforms. Such a window of opportunity argument is first offered by A. Krueger (1993) when drawing on the reform experiences from reforming countries (mainly Latin American countries) in the 70's. President Yeltsin's charisma and popularity together with optimism among contemporaneous Russian thinkers and reformers led to a rare opportunity for the Russian government to adopt drastic reform.

A premise of our theory is that the young are more forward looking, comparing different policy outcomes when deciding their positions. Hence, when the government shows no interest at all in implementing reform, a young population structure is more impatient and more susceptible for revolts than an old population structure is. As a young population structure is compatible to both being too patient and being too impatient, this posts challenges in the empirical testing of our theory.

Whereas the economic reforms of China and Russia are just too complicated to be fully explained only by demographic factors, we hope our approach is complementary to existing approaches in understanding the issues. Our general goal is to present a framework that understands the long Chinese reform as an intergenerational game in which the structure and dynamic of demography play a role in it.

The rest of the paper is organized as follows. Section 2 presents the baseline model in which there is only one reform strategy which we call gradual reform. Section 3 solves the model, focusing the young workers' support of reform and how the support varies with the number of children each young worker has or is expected to have. Section 4 continues to solve the model by characterizing the dynamic equilibrium. Section 5 introduces the option of having a big bang reform and studies the government's choice between big bang and gradual reform. Section 6 discusses some key assumptions and reviews the related literature. Section 8 concludes.

2 Baseline Model

We first provide a description of the model, followed by justification of the main assumptions.

2.1 Description

We consider a discrete-time infinite horizon model. Each agent lives for four stages (periods) as stage-0 child, stage-1 young worker, stage-2 middle-aged worker, and stage-3 retiree. A stage-0 child depends on her parents for consumption and makes no decisions (we always use female pronouns to refer to agents in this paper). A young worker works and bears children and raises them until they become young workers in the next period. A middle-aged worker works, but does not need to bear or raise children, because her children have already grown up. A retiree does not work and lives on her previous savings. Agents from an earlier stage move on to the next stage with certainty except that retirees will die at the end of their retiree stage.

A young worker's life time utility is

$$U = A(n) + u(c_1) + \delta u(c_2) + \delta^2 u(c_3), \qquad (1)$$

where c_i is her stage-*i* consumption, i = 1, 2, 3 and *n* is the number of children the young worker has; for simplicity, we assume that *n* is exogenously given⁴ (even though it may not be the same from period to period). The stage utility function in each stage *i* takes the form of constant relative risk aversion (CRRA), i.e.,

$$u(c_i) = c_i^{1-\rho} / (1-\rho),$$

where $\rho > 0$ and is not equal to unity (when $\rho = 1$, the stage utility function is replaced by $u(c_i) = \ln(c_i)$). Notice that $1/\rho$ is the constant elasticity of intertemporal substitution.

In this formulation, the young parent derives utility from having more children, but does not derive utility from her children being fed more. (This approach is used in a strand of

⁴see a justification in section 7.

literature on population and economic growth, see, e.g., Galor and Weil (1996).) Despite this, she feels obliged to feed them—the more she budgets for own consumption, the more she will budget for her children's consumption. More specifically, when a young worker decides to consume c_1 in that period, she budgets a total of $\gamma(n) c_1$ for her children's consumption. We assume that $\gamma(n)$ is differentiable and strictly increasing in n. Thus the more children, the more needed to be fed to them. Parents no longer support children once the latter have grown up, nor do grown-up children support their parents. Finally, while agents can always save their income, at a fixed interest rate of r, we assume they are not allowed to borrow. (Allowing them to borrow at a fixed interest rate exceeding r will not alter the qualitative nature of our results, however.)⁵

2.2 Reform

At the status quo, each worker's production is x in each period. The economy suffers from a system-wide inefficiency. A reform, which takes one period to complete, increases the output per worker to y > x; however, during the reform period, the output per worker is y - k only, where k is a loss, known as the reform $\cos k$, to be incurred by every worker. The loss may be the effort required to learn a new skill, hardship coming from adoption to a new, harsher environment, etc. Here we highlight the investment nature of reform (in the spirit of Krusell and Rios-Rull 1996); while benefiting in the long run, it is costly in the short run. We assume that neither children nor retirees are directly affected during the reform period.⁶ Finally, the completion of reform is subject to public approval through simple majority voting (SM) (we will also discuss, when appropriate, the rule of unanimity where transfers are allowed (UT)). Notice that the voting should not be understood literally; it is simply meant to capture the idea that reform requires popular support. In the baseline model, the role of the government is simply to propose the reform at the beginning of each

⁵Because our study is set at a plan economy like China in late 70's, it makes sense to assume poor financial markets and closed economy in which no international lending and borrowing are allowed. While Eastern European countries did have borrowing from western countries in 70's, they also faced difficulties in borrowing more in late 80's (in fact, Poland and Russia were both highly indebted at that time and had difficulties to borrow more).

⁶If retirees are indeed affected, they are likely to be affected negatively. Thus a population with a larger fraction of retirees makes gradual reform less likely and our argument will hold more easily.

period until it is adopted. In a later extension, we allow the government to play a more active role as agenda setting — to propose a big bang or a gradual reform—and we will discuss the government objective function more thoroughly there.

Figure 1 is a summary of the sequence of events at period t when a reform proposal is contemplated. At time t.1, the children in the last period turn young workers, the young workers in the last period turn middle-aged workers, and the middle-aged workers in the last period turn retirees. At time t.2, the young workers each give birth to n_t children, where n_t is exogenously given, perfectly foreseen. At time t.3, agents "vote" on the reform.⁷ At time t.4, given the reform decision, workers receive their income, consume, feed their children (if they are young workers), and save; retirees get back their savings, consume, and finally die. Then the surviving agents move on to the next period t + 1. If reform is not successfully adopted in period t, it will be proposed again in period t + 1, following a similar sequence as depicted in Figure 1.

We emphasize the importance of public support to reform even for an autocracy like China.⁸ Although an autocracy may have more muscle than a democracy, there are still limitations —- it cannot force the public to make wise, risky business decisions or to be innovative, etc. Modern incentive theory supplies additional arguments. For instance, an autocrat's determination to reform may be unknown to the public (adverse selection problem); or the autocrat may have difficulty refraining itself from expropriating private investment ex post (moral hazard problem). Time is often the ultimate solution to these problems,⁹ and lacking popular support in a long period of time may jeopardize the resolution of these

problems.

⁷Given that the fertility rate is exogenous given, the order of voting and giving births can be switched without affecting the result.

⁸This is supported by the emphasis of notions such as Pareto improvement reform or reform-withoutlosers motivated by the Chinese experience (see Lau, Qian, and Roland 2000 and Qian, Roland, and Xu 2006).

⁹It will require a sufficiently long period of time for the autocrat to credibly signal its "type" (strong resolution versus weak resolution); the reform may need to take a gradual approach so as to mitigate the commitment problem.

It is not difficult to find evidence of both problems in China. There was clearly widespread concerns during the period of 1989-1992 about the future of reform. Deng Xiaoping's visit to southern coastal cities in 1992 is commonly viewed as a decisive boost to, determination of, the continuation of reform. These events can be seen as supportive of the adverse selection model. It is worthwhile noting that, while Chinese reform started in 1978, the FDI into China did not surge until early 2000s.



Figure 1: Event schedule in period t, when a reform proposal is made and decided.

3 Young workers' attitude towards reform

In this section, we examine the public's attitude to reform at time t.3. We assume their attitude is aggregated using simple majority voting (SM) without transfers of income. Since the retirees are not affected by the reform, we assume they do not vote at all. We use $\alpha \geq 1$ to denote the weight given to a vote by the middle-aged relative to a vote by the young. For each middle-aged worker, because her period income is higher under no reform (equal to x) than under reform (equal y - k < x), she will vote against reform. For each middle-aged worker, there are n_{t-1} young workers. If $n_{t-1} < \alpha$, the middle-aged workers are numerous enough to block reform. Thus, one necessary demographic condition for reform to be passed is that $n_{t-1} \geq \alpha$, i.e., the young-workers-to-middle-aged-workers ratio must exceed a critical number.

3.1 Young workers' support of reform

3.1.1 Utility maximization

We now turn to young workers' voting decisions. Given generic first-period and secondperiod incomes z_1 and z_2 , a representative young worker's utility maximization problem is as follows:

$$\max_{c_{1},c_{2},c_{3}} u(c_{1}) + \delta u(c_{2}) + \delta^{2} u(c_{3})$$

subject to intertemporal budget constraints (i) $(1 + \gamma(n))c_1 + s_1 = z_1$, (ii) $c_2 + s_2 = z_2 + (1+r)s_1$, and (iii) $c_3 = (1+r)s_2$, and non-negativity of c_1, c_2, c_3, s_1 , and s_2 , where s_1 and s_2 are savings in stage-1 and stage-2, respectively. (The component A(n) is omitted from the utility function because n has been realized.)

It is easy to verify that consumption smoothing is feasible if z_1 is sufficiently large relative to z_2 . The optimal solution to the problem satisfies the following two FOCs

$$u'(c_1) = \delta(1+r)(1+\gamma)u'(c_2)$$
(2)

$$u'(c_2) = \delta(1+r) u'(c_3),$$
 (3)

and stage-1 consumption c_1 is given by

$$c_1 = \frac{1}{1 + \gamma + E} \left(z_1 + \frac{z_2}{1 + r} \right).$$
(4)

where

$$E \equiv \frac{\left(\delta\left(1+r\right)\left(1+\gamma\right)\right)^{\frac{1}{\rho}}}{1+r} + \frac{\left(\delta^{2}\left(1+r\right)^{2}\left(1+\gamma\right)\right)^{\frac{1}{\rho}}}{\left(1+r\right)^{2}},\tag{5}$$

When consumption smoothing between stage 1 and later stages will not be feasible, the optimal solution is modified as follows: (i) the equality sign in (2) will be replaced by a strictly greater than sign; (ii) consumption smoothing between the stage 2 and stage 3 will still be feasible, i.e., (3) continues to hold true; and (iii) equation (4) is replaced $(1 + \gamma) c_1 = z_1$, i.e., all income z_1 is spent for the period's use.

3.1.2 Greatest endurable hardship

Now that we have solved the young worker's utility maximization problem, we are ready to study her reform support decision. If reform is approved, her current-period and next-period incomes are $z_1 = y - h$ and $z_2 = y$, respectively, where h is the hardship inclusive of any transfer-out payment (i.e., h = k under the simple majority rule and $h = k + \Delta$ under the unanimity-with-transfers rule where Δ is the transfer to the middle-aged workers).¹⁰ If the reform is not passed, the current period income is x and the next-period income, denoted by B(h), which we call the post-delay income, depends on what will happen in the next period (in particular whether or not a reform will be implemented then) and should be endogenized in a fully dynamic game, which we relegate to Section 5. Here, we simply posit it by a general function with the following properties.

A1 (i) $0 \le B(h) \le y$; and (ii) $-1 \le B'(h) \le 0$.

Point (i) states that B(h) cannot exceed what a worker will earn subsequent to the completion of the reform. Point (ii) states that B does not decrease in h as quickly as does the current income under reform, which is equal to y - h. This formulation accommodates three interesting cases:

- B(h) = y h: the reform is approved next period;
- B(h) = x: the reform is not approved next period;
- B(h) = y L, where L is fixed and larger than any conceivable h; it captures a more disruptive change

We use $U_R(h, n)$ to denote the young worker's resulting indirect utility under reform and $U_D(h, n)$ her indirect utility function under delay, where n is the number of children she

 $^{^{10}}$ The reader may simply construe *h* to be *k* if he or she finds it easier to focus exclusively on the case of SM, which is also the focus of the paper.

has. We first show there exists a unique greatest endurable hardship h^* such that, at time t.3, the young worker prefers having reform now to having it delayed if and only if $h \le h^*$. (Proofs to Lemmas and Propositions are relegated to the appendix unless otherwise stated.)

Lemma 1 Given assumption A1 and n, there exists $h^* \in (y - x, y)$ such that for all $h < h^*$, $U_R(h, n) > U_D(h, n)$; for all $h > h^*$, $U_R < U_D$.

The result is intuitive. We first notice that both U_R and U_D are decreasing in h. However, U_R decreases at a greater rate than U_D does because of assumption $A1.^{11}$ Hence, if U_R and U_D are ever equal at some h, this h must be unique (denoted by h^*) and $U_R \stackrel{>}{\leq} U_D$ if and only if $h \stackrel{\leq}{\leq} h^*$. It is easy to check that h^* is in between y - x and y.

We now study how h^* varies with the exogenous n. Totally differentiating $U_D(h, n) = U_R(h, n)$ and rearranging terms, we obtain

$$\frac{dh^*}{dn} = \left(\frac{\partial U_D}{\partial n} - \frac{\partial U_R}{\partial n}\right) / \underbrace{\left(\frac{\partial U_R}{\partial h} - \frac{\partial U_D}{\partial h}\right)}_{-ve}$$
(6)

Whereaus the denominator of the RHS term is positive, the sign of the numerator can be found out using the following Lemma.

Lemma 2 For regime j = R (reform), D (delay),

$$\frac{\partial U_j}{\partial n} = A'(n) - \frac{\gamma'(n)}{1 + \gamma(n)} c_1^j u'\left(c_1^j\right),\tag{7}$$

whether or not consumption smoothing is feasible between period 1 and later periods.

(7) is interpreted as follows. Consider the case where consumption smoothing is infeasible under regime j. Whereas the first term in the RHS of (7) is the direct effect of having one more child, the second term is the indirect effect due to a lowering of consumption (the young worker will spend all her current income in that period). The term, $\frac{\gamma'(n)}{1+\gamma(n)}$, measures the additional fraction of the parent's consumption that is required to raise an additional child.

 $^{^{11}}h$ appears in the second period income under delay while in the first period income under reform. Due to discounting and the opportunity of savings, an equal change of h has smaller impacts on the indirect utility under delay.

The second term $c_1^j u' (c_1^j)$ is the change in the worker's period-1 utility given one unit change in $c_1^{j,12}$ Lemma 2 states that (7) holds true even if consumption smoothing is feasible. The intuition is that in the consumption smoothing case, equalization of discounted marginal utility across periods will ensure that the change in the life-time utility can be represented in terms of the change in the first period's utility and hence (7) remains to be true.

Given Lemma 2, (6) is easy to sign. Substituting (7) into it, we obtain

$$sign \left| \frac{dh^*}{dn} \right| = -sign \left| \frac{\partial U_D}{\partial n} - \frac{\partial U_R}{\partial n} \right|$$
$$= +sign \left| c_1^D u' \left(c_1^D \right) - c_1^R u' \left(c_1^R \right) \right|.$$
(8)

The young worker is more likely to face difficulty in consumption smoothing under reform than under delay.¹³ The equality of U_R and U_D is thus achieved either by (i) $c_1^R < c_1^D, c_2^R > c_2^D$ and $c_3^R > c_3^D$ (consumption smoothing infeasible under reform) or by (ii) $c_i^R = c_i^D$ for i = 1, 2, 3 (consumption smoothing feasible under reform).

In the former case, provided $\rho > 1$, $c_1^R < c_1^D$ implies $c_1^D u'(c_1^D) - c_1^R u'(c_1^R) < 0$ and an additional child reduces the agent's utility under reform more than it does under delay and $dh^*/dn < 0$ (the sign will be reversed if $\rho < 1$). In the latter case, since $c_1^R = c_1^D$, an additional child reduces the agent's utility under reform as much as it does under delay, and $dh^*/dn = 0$ regardless of ρ . This leads to our first main result.

Proposition 1 Given the post-delay income B(.) that satisfies A1, there exists a critical n^* such that (i) for $n < n^*$, h^* is independent of n; and (ii) for $n \ge n^*$, h^* decreases with n if $\rho > 1$ and increases with n if $\rho < 1$.

Note that n^* is the number of children such that the young workers' consumption smoothing condition just becomes non-binding given that $U_R = U_D$. A natural question is how large n^* is. If we take the view that in practice young parents do have difficulty in smoothing

 $^{{}^{12}}c_1^j u'\left(c_1^j\right)$ can be re-written as $(1-\rho) u\left(c_1^j\right)$. For $\rho > 1$, $u\left(c_1^j\right)$ is negative. When c_1^j is bigger, $u\left(c_1^j\right)$ is less negative and $(1-\rho) u\left(c_1^j\right)$ is smaller.

¹³Given that $h^* > y - x$, the income profile under reform is more back-loaded than its counterpart under delay is, i.e., the ratio of current-period income over next-period income under reform, (y - h)/y, is smaller than its counterpart under delay, x/B(h).

consumption, then we can conclude that n exceeding $n^*(B)$ is indeed the relevant range to focus on.¹⁴ It is interesting to notice that the quantitative macro literature suggests that ρ is most likely to exceed unity. Given these two observations, we have the following conclusion. At time t.3, given the post-delay income function B, a young worker's greatest endurable hardship is negatively related to the number of children she has. Before moving to the next section, we state a simple result for future use.

Lemma 3 Assume h < y - x. Let \overline{h} be the h^* solved assuming B(h) = y - h and \underline{h} be the h^* solved assuming B(h) = x. Then $\overline{h} > \underline{h}$.

Lemma 3 compares two future contingencies. In the first, the delayed reform will be approved in the next period; in the second, it will not be approved in the next period either. The lemma states that the young agent's greatest endurable hardship under the first contingency is greater than under the second. That is, relative to the prospect of further delay of reform, the threat of having the delayed reform implemented in the next period makes the young agent more likely to accept it now. This result is useful for us to understand how the post-delay income is endogenously determined.

4 An alternative formulation of child rearing cost

Here we provide an alternative formulation of the costs of having children. Assume that, to raise *n* children, there are two costs to their young-worker parent: a fixed consumption cost of *T*(*n*) dependent on the number of children and a fraction of the parent's time κ (*n*) being used up, where both *T*(*n*) and κ (*n*) are differentiable and increasing in *n*.¹⁵ As a result, the young-worker parent obtains an stage-1 income of $(1 - \kappa(n))x$ under the status quo and a stage-1 income of $(1 - \kappa(n))y$ under the period of reform. After netting the reform hardship, her stage-1 income under reform is $(1 - \kappa(n))y - h$ only. Define indirect utility functions as a function of *h* and *n* only, i.e., $U_R(h, n)$ and $U_D(h, n)$. It is straightforward to

¹⁴ It can be shown that when r = 0, $n^*(B) = 0$ in the case that B takes on the specification of B(h) = y-h or y - H.

 $^{^{15}{\}rm See},$ for instance, Becker and Barro 1988 and Galor and Weil 1996 on modeling cost of having children as time cost.

show that Lamma 1 still holds in this context. That is, h^* exists such that $U_R \geq U_D$ if and only $h \leq h^*$ and $\frac{\partial U_R}{\partial h} - \frac{\partial U_D}{\partial h} < 0$. Using (6), we obtain $sign \left| \frac{dh^*}{dn} \right| = -sign \left| \frac{\partial U_D}{\partial n} - \frac{\partial U_R}{\partial n} \right|$.

Assume for simplicity the consumption smoothing condition is binding in both policy regimes. A unit increase in n hurts the young parent because, besides putting aside an extra amount of consumption T'(n) for her children, she also works less, suffering an income drop of $\kappa'(n) z_1^j$, where $z_1^R = y$ and $z_1^D = x$. The change in her life time utility under regime j is

$$\frac{\partial U_j}{\partial n} = A'(n) - T'(n) \, u'\left(c_1^j\right) - \kappa'(n) \, z_1^j u'\left(c_1^j\right). \tag{9}$$

Besides the term A'(n), there are two components to the change and both are negative. The first term, due to the increased consumption by children, is more damaging under reform than under delay because $c_1^R < c_1^D$. The second term, due to a reduction of time spent on work, is also more damaging under reform than under delay; the reason is that the reduction of income because of an extra child is greater under reform (due to higher income) and a reduction of a unit of consumption reduces utility more under reform (due to a lower consumption level). Therefore, we can show that $\partial U_D/\partial n - \partial U_R/\partial n > 0$ without any restriction on the value of ρ . Notice that the fixed consumption part is not essential to this result and that the above argument holds true even if consumption smoothing between stage 1 and later stages is feasible. We summarize this result in the following proposition.

Proposition 2 Suppose the child rearing cost involves time cost plus possibly a fixed consumption and the post-delay income B(.) satisfies A1. The greatest endurable hardship h^* is decreasing in n, regardless of ρ .

We have assumed that the reform hardship h applies to every young worker, regardless of the fraction of time worked. One interpretation is that it arises from learning a new skill, and a certain degree of proficiency is needed whether the worker wants to work full time or part time. Of course, if the hardship is proportional to the fraction of time spent on work, the analysis will be different.

5 Aggregate decision making

In previous sections, we characterized the greatest endurable hardship (*GEH*) assuming a post-delay payoff function B(.). In this section, we clarify how the *GEH* that can be supported in equilibrium is determined and whether our earlier insights—regarding the roles of current fertility rate and demographic structure—still hold. In the fully dynamic game, there are usually multiple equilibria, each being associated with an equilibrium greatest endurable hardship (*EGEH*). Among all the *EGEH*s, there is a maximum one, which we call the maximum equilibrium greatest hardship (*MEGEH*).¹⁶ To fix ideas, we also assume the use of simple majority voting (SM) so that the hardship from the reform during the implementation period is always k (nonetheless, we still keep the use of h, and in this case h = k).

It is easy to see that the *MEGEH* at period t, denoted by h_t^m , must be bound above by \overline{h}_t and below by \underline{h}_t , which we recall are the *GEH* of the representative young worker based on her belief that in the next period the delayed reform will be approved and will not be approved, respectively. It is also easy to see that, if this former belief is consistent with some equilibrium, then h_t^m is indeed equal to \overline{h}_t . Otherwise, it must strictly be less than \overline{h}_t , and may be equal to or strictly exceed \underline{h}_t . The latter point being more intrigue, we will illustrate it through a particular example.

Assume that (i) $n_{t-1} > \alpha$, (ii) all future fertility rates $n_{t+i} = n^*$, where i = 1, 2, ...,and (iii) $n^* > \alpha$. The second assumption ensures a constant young-workers-to-middle-agedworkers ratio (n^*) in future periods t+2, t+3, ... The first and third assumptions ensure the young workers in period t, as well as those in period t+2 and onwards, are numerous enough to overwhelm their middle-aged worker counterpart in voting. We depict h_t^m in Figure 2, where the horizontal axis is n_t and the vertical axis is h. $\overline{h}(n_t)$ and $\underline{h}(n_t)$ are the *GEH*s defined and solved in Lemma 3, and they are downward slopping because of Proposition 1 and the assumption that $\rho > 1$.

There is a tripartite classification of n_t : (i) $n_t < \alpha$; (ii) $\alpha \le n^t < n^*$; and (iii) $\alpha < n^* \le n^t$.

¹⁶ There is a corresponding notion of minimum *EGEM*, which we will not explore.

Case i: $n_t < \alpha$. The current period's fertility rate is so low that the current young workers are numerous enough to dominate their children in next period's voting. In other words, if reform is not approved in period t, it will not be approved in period t + 1 either. Thus h_t^m is simply $\underline{h}(n_t)$.

For the other two cases where $n_t > \alpha$, the young workers will not be numerous enough to overwhelm the next generation in next period's voting. Note that for period t + 2 and onwards, the ratios of young workers to middle-aged workers are constant, equal to n^* ; we can use $\overline{h}(n^*)$ and $\underline{h}(n^*)$ to denote the *GEH*s in periods t + 1, t + 2, ... based on the belief that the reform will be and will not be approved, respectively, in the subsequent period.

Case ii: $\alpha \leq n^t < n^*$. Note that any reform with h strictly greater than $\overline{h}(n^*)$ will not be accepted at period t + 1 if the reform is voted on that period. Since $\overline{h}()$ is decreasing in its argument, we have $\overline{h}(n^*) < \overline{h}(n_t)$ and h_t^m must be less than $\overline{h}(n_t)$. Hence, h_t^m is equal to $\underline{h}(n_t)$ if $\underline{h}(n_t) \geq \overline{h}(n^*)$ and equal to $\overline{h}(n^*)$ if the reverse is true. In other words, h_t^m is equal to max $\{\overline{h}(n^*), \underline{h}(n_t)\}$.

Case iii: $\alpha < n^* \le n^t$. Because $\overline{h}(n_t) < \overline{h}(n^*)$, the argument in case (ii) that "kills" $\overline{h}(n_t)$ as h_t^m no longer works. Therefore, h_t^m is simply $\overline{h}(n_t)$.

We summarize the above discussion as follows.

Proposition 3 Suppose (i) $n_{t-1} > \alpha$, (ii) all future fertility rates $n_{t+i} = n^*$, where i = 1, 2, ..., and (iii) $n^* > \alpha$. Then

$$MEGEH = \begin{cases} \underline{h}(n_t) & \text{if} \quad n_t < \alpha \\\\ \max\left\{\underline{h}(n_t), \overline{h}(n^*)\right\} & \text{if} \quad \alpha \le n_t < n^* \\\\ \overline{h}(n_t) & \text{if} \quad n_t \ge n^* > \alpha \end{cases}$$



Figure 2: The *MEGEH* as a function of current fertility rate n_t and future fertility rates n^* . The two panels differ in the value of n^* . An increase in n^* lowers both $\overline{h}(n^*)$ and $\underline{h}(n^*)$, as well as the *MEGEH* profile.

Some general lessons can be drawn. First, other than some small qualifications,¹⁷ the MEGEH is indeed decreasing in n_t and our analysis is supportive of the insight from Proposition 1. Second, the initial population structure may be pivotal in determining the popularity and feasibility of the reform. In the above example, if we invoked assumption (i) so that $n_{t-1} < \alpha$, then the reform cannot be approved in the current period because middle-aged workers are too numerous, and this is so even if the current young workers have low fertility rate which is presumably favorable to reform. Third, fertility rates in the far future may have impacts on the current MEGEH because of backward induction. The two panels in Figures show that a lowering of future constant fertility rate n^* (switching from panel b to panel a) leads to a higher MEGEH for the current period.¹⁸

6 Reform strategy choices

It is time to interpret our results in terms of a choice of reform strategies. Thus far we have discussed the feasibility of a generic reform which is completed in one period and is subject to popular approval/support. Our intended interpretation of the reform is that it is a gradual reform subject to trials and errors and is, therefore, reversible upon popular dissatisfaction. However, as the inner structure and scheduling of different components of the reform package are assumed to have been ideally solved and determined, the gradual nature of the reform is oblivious to the reader. We also assume that this reform is optimal (compared to all other alternatives); we maintain that because of the difficulty in transition to market economy from plan economy, the reform must have some gradualism charactistics. Our analysis has shown that such an ideally designed package may not be feasible under unfavorable demographic conditions.

Now imagine that there is an alternative reform strategy: big bang strategy which, while also taking one period to complete, is irreversible once started. By construction, the

¹⁷It is so except for the neighborhood when n_t is equal to α (as in panel a of Figure 2). The potential non-monotoncity happens because of a shift of h_t^m from max $\left\{\underline{h}(n_t), \overline{h}(n^*)\right\}$ to $\underline{h}(n^*)$ when n_t moves across α .

¹⁸This is consistent with the fact that not until very recently (after four decades of one-child policy!) has the Chinese government indicated signs of relaxing its birth control.

big bang reform must be less efficient (taking the form of larger short-term hardship, for example) so that a benevolent government will not choose it under favorable demographic conditions. In the absence of such favorable demographic conditions, the government will face perennial opposition when gradual reform is attempted and hence may choose big bang. This thus rationalizes why with the benevolent government, gradual reform could be chosen in a country like China and big bang reform could be chosen in a country like Russia.

Notice that even if demography does not allow a gradual reform, it may allow a big bang to take place under exceptionally situations, where a "window of opportunity" (during which either the government is unconstrained, or the public is more lenient than they should be in hindsight) exists that may allow a big bang to be pushed through.¹⁹ The "window of opportunity" argument is first offered by A. Krueger (1993) when drawing on the reform experiences from reforming countries (mainly Latin American countries) in the 70's.

Conceivably, such a "window of opportunity" was present in Russia in early 1990s' when the government was relatively unconstrained in adopting drastic reforms. President Yeltsin was charismatic and popular. Russian reformers and their economic advisors at the time were optimistic about market efficiency. As famously acknowledged by Milton Friedman, "only a crisis—actual or perceived—produces real change. When that crisis occurs, the actions that are taken depend on the ideas that are lying around." (Friedman and friedman, 1982 preface, p. ix) These two factors combined led to a rare opportunity for the Russian government to adopt drastic reform.

Admittedly, the distinction between gradual reform and big bang in our framework is stark. They differ in reversibility, but not in an explicit modeling of different durations of completion time.²⁰ A big bang is a commitment in the sense once many aspects of reform (from privatization, labor market liberalization, private ownership, free banking, floating exchange rate, free capital flows, etc.) have been taken speedily and simultaneously, it will

¹⁹This is reminscent of the following famous quote, "You can fool all the people some of the time, and some of the people all the time, but you cannot fool all the people all the time."

 $^{^{20}}$ As one period in our model means about one fourth of a life time, say, 15 to 20 years, the fact that a reform is completed in one period is compatible to both interpretations of gradual reform and big bang reform. Moreover, one interpretation is that the big bang reform takes a fraction of a period to complete, while the gradual reform takes the whole period to complete.

be difficult to revert. The view that big bang reform involves greater reversal costs is not new in the literature. As pointed out by Dewatripont and Roland 1995, p.1208), "[a] bigbang strategy involves high reversal costs, which are often considered to be an advantage ex post since it reduces the reversibility of enacted reforms, which is a constant concern for reformers."²¹

Many pros and cons regarding the relative merits of gradualism and big bang have been pointed out.²² Here we can only discuss a few most related work. Dewatripont and Roland (1992a,b) and Wei (1997) find that, when a reform is thwarted by political constraints, gradualism—as a divide-and-conquer strategy—may soften the constraints and may go through despite some compromise in speed or efficacy. While forcing the policy to be conducted in a longer duration in their framework, political constraints force the policy to be hastened in our framework. Despite the difference, both lines of studies argue that political constraints may lead to a distortion in policy implementation.²³

Motivated by the Soviet Union and Eastern and Central European countries' experience, Blanchard and Kremer (1997) provide a model to explain why there is reduction in production when a plan economy is moving towards market economy. The problems arising from incomplete contracts and asymmetric information that can be mitigated either through plan economy or fully fledged market economy are the worse during economic transition. Relatedly, Li (1999) presents a model of transitional economy that features initial output reduction under the big bang reform but output increase in Chinese-style reform. The key is that in the former the dismantling of central planning allows monopolistic enterprises to gain from restricting in output while the Chinese-style reform explicitly requires enterprises to fulfill its planned quotas first. Murphy, Shleifer, and Vishny (1992) emphasize the commitment role of big bang which averts a partial reform trap that would otherwise result from

 $^{^{21}}$ The importance of commitment over discretion in policy making is first famously made by Kyland and Prescott (1976). In spirit, our emphasis of big bang reform as a commitment device is similar to the use of extension of the franchise to the poor by the elite so as to commit to future re-distribution policy in Acemoglu and Robinson (2000).

 $^{^{22}}$ For instance, in its introductory section (p.1235), Wei (1997) lists out six reasons (!) in favor of big bang and four reasons (!) in favor of gradualism.

 $^{^{23}}$ Che (2007) is concerned about the timing of privatization and the expost performance of privatized firms. Government ownership is more efficient than private ownership when private property rights are insecure. As institutional protection of property rights is improving over time, there is a need to privatize. But the buyer's financial constraints affect its timing, hence affecting the firm's post-privatization performance.

gradual reform. The article by Qian and Xu (1993), which we introduced earlier, argue that China, despite its planned economy, was organized quite differently. Its M-form organizational structure, rather than the U-form in the former Soviet Union, allowed it to adopt experimentation and make changes gradually (See also Qian, Roland, and Xu (2006)).

7 Discussions

7.1 alternative voting mechanism

In Sections 2 to 5, we examined the effects of demography on the feasibility of a generic reform using a simple majority rule without transfers when agents make collective decisions. Here we argue that similar ideas hold true if we assume middle-aged workers each has veto power so that young workers must make enough transfer to them as compensation (we call this voting mechanism as unanimity voting with transfer of income, UT).

As each middle-aged worker suffers an income loss of x - (y - k) under reform, compared to under no-reform, for the reform to go through, each young worker makes a transfer of

$$\Delta \equiv \left(x - y + k\right) / n_{t-1}.\tag{10}$$

The total hardship each young worker endures is thus equal to $h = \Delta + k$ under reform (her net income in the period being equal to y - h). As Δ is decreasing in n_{t-1} , so is h. Regarding the role of current fertility rate (n_t) , note that in subsection 3.1, where we studied the maximum endurable hardship for young workers, we did not make use of any voting mechanism. Therefore, the result obtained there—that h is decreasing in n_t —continues to hold true under the alternative voting of UT.

In summary, our results are consistent with our early findings under SM over which (i) the higher the young worker-to-middle-worker ratio, the more favorable it is in supporting reform (under SM, the effect is more stark: the ratio must be greater than α for the reform to gain enough support) and (ii) the lower the current fertility rate, the higher the maximum endurable hardship the young workers have.

7.2 exogenous fertility rate and support of birth control

In the analysis, we have assumed exogenous fertility rates. In the case of Russia, the favorable role of the decreasing number of children may be dominated by the unfavorable old population structure, and assuming exogenous fertility role is a benign simplification. In the case of China, the one child policy can be viewed as a binding constraint, and hence the number of children is an exogenous variable in young couples' decision making.

Despite our stress on the importance of public support to economic reform, we have not modeled the public support to birth control policy. One reason is that the necessity of support is less important. Relative to the noncompliants of economic reforms that are difficult to detect (non-observable efforts, uncertain outputs, team production, etc.), the noncompliants of birth control are easier to identify, to punish ex post, or even to deter ex ante.

Moreover, the policy may not be as disagreeable as it appears to be, for several reasons. Given that demographic transition has been a global phenomenon, it is just a matter of time when Chinese would significantly reduce their fertility rates; fertility choices depend on the prevalent social norm (Munshi and Myaux 2005) and the one-child policy has simply hastened the shift of norm. The public might indeed agree that, in the absence of birth control, there are more births than socially optimal because fertility exerts negative externality (see Johnson 1974). In light of Chinese's son preferences, the availability of gender selection technology since the mid 1980's has allowed Chinese households to experience a reduction in the number of sons that is less restrictive than literately implied by the one-child policy.

7.3 government objective

We have not specialized the government's objective. In the main model where there is just one generic reform, we have assumed that the government will continue to propose the generic reform each period until it gets enough support from the public. Given that (1) the reform increases productivity and (ii) there is only one reform policy available for choice (in particular, fertility rates are exogenous and not choice variables and big bang reform is not an option), the aforementioned government behavior may well be consistent with the objective a benevolent government. If birth control is also a policy choice, i.e., population size is also a choice valuable for the government, the government is likely to value both higher total national income and higher national income per capita (for both current and future). For a benevolent government, lowering the current fertility rates (even future fertility rates) to make an infeasible gradual reform to become feasible may not attractive. Finally, given unfavorable demographic conditions, adopting big bang in light of a blocking of gradual reform may also be consistent with a benevolent government assumption. Generally, all the discussions should go through as long as the government's objective is not too far away from benevolence.

7.4 what officials think of birth control?

That birth control is helpful to the economy in a political economy sense can also be found from government officials' speeches. A provincial leader, for example, announced in 2009 that due to the one-child policy in the last 30 years, "the province has cut down the number of births by 10.6 millions and hence has increased the per-capita GDP by RMB2,944 and contributed to one fourth of the economic social development". (source...searched on...)

The ideas seem to be as follows. A new born baby is not ready for labor market participation until 15 or 20 years later. Therefore, in the first 15 to 20 years of the policy's inception, while not the addition of a single worker to the labor force was prevented, a lot of burdens to families and society were avoided. This thus led to an increase in the per-capita GDP (compared with the case where one-child policy is not enforced), not to mention the additional effects of having a greater supply of female labor because of less time devoted for motherhood.

This simple math may have significant implications. A family is easier to feed its members above the subsistence level, and poverty rate is reduced without any change to production and income distribution. Suppose young people support government's initiatives as long as their expected living standard does not fall below a reference point, and that they consider their reference point to be their parents' living standard during the former's childhood or their expected living standard *under no reform and no one-child policy*. Then one-child policy makes such a reference point easier to reach. In this paper, however,²⁴ by taking a more standard, a harder approach regarding the choice of support, we have arrived at conclusions that are close to what the government official intended to make.

7.5 population

The literature on population is too vast to summarize here. We are contented to review some papers on Chinese population and economic reform. In a series of papers, Wei and his coauthors study the impacts of sexual imbalance in China. According to these studies, sexual imbalance (having more boys than girls) might lead to higher saving rates (Wei and Zhang, 2011) and trade surplus (Du and Wei 2013). Using Chinese data, Li and Zhang (2007) find out that birth rate has a negative impact on economic growth, suggesting that Chinese one-birth policy is conducive to economic growth. Liao (2013) studies the effect of one child policy on labor market. However, there is political economy in the model and no policy needs to be made.

The political economy of demography is well studied in pension policy and immigration policy. These are where intergenerational conflicts are conspicuous (see, e.g., Sand and Razin 2007 and Storesletten 2000). However, as well as we know, the possibility that a change in the fertility rate may play a role in the game has not been studied.

8 Conclusions

Our paper is motivated by the stark difference in the demographic structures in Russia and China at the beginning of their reforms. We first pointed out that without taken into account of demography some key features of the China (sustained growth of both the nonstate sector and state sector) would not be feasible and that Russia would not bear the features simply because of their shrinking population size.

 $^{^{24}}$ This history-based reference point is plausible. Chinese people are often remained of their miserable life before 1949. The practise serves as a "prime" so that the current hardship becomes more endurable. The healing, psychological effect is neatly summarized by a popular phase: "yikusitian", literally "remembering the bitterly past, appreciating the sweetly present."

We have presented a simple theoretical model in which whether the gradual reform can be adopted depends on the demography of the economy. Besides the result that an old population structure lends less support to the reform, we have also found that the representative young worker's attribute towards reform is more positive—the maximum endurable hardship is higher— when she has or is expected to have few children. This thus suggests some subtle political economy implications of the one-child policy.

We have shown that, if the demographic conditions are unfavorable and the gradual reform cannot be adopted, then a benevolent government may want to hasten the reform to make it irreversible, whenever such a window of opportunity exists.²⁵ This thus provides a unified theory justifying that, in principle, both the big bang in Russia and gradualism in China could be optimal given each respective country's specific conditions.

Our key emphasis is that we understand Chinese reform as an inter-generational game. Given the long reform process, we think an inter-generational framework shall be a useful one. That said, our model is very simplistic. We do not explain the high economic growth rate,²⁶ nor do we argue that other factors that have been identified in the literature (specific institutions, the role played by Deng Xiaoping, large rural population, etc.) are not essential.

A premise of our theory is that the young are more forward looking, comparing different policy outcomes when deciding their positions. Hence, when the government shows no interest at all in implementing any reforms, a young population structure is more impatient and more susceptible for revolts than an old population structure is. As a young population structure is compatible to both being too patient and being too impatient, this posts challenges in the empirical testing of our theory.

²⁵When such a window of opportunity does not arise, however, the country with unfavorable demographic conditions may come to halt in terms of reform. This possibility is not highlighted in the paper. Moreover, our paper also does not offer a theory when a window of opportunity would exist.

 $^{^{26}}$ Our model exhibits only one time growth in per capita productivity rather than continous growth. This is one shortcoming of our paper. For a paper that explains its high growth rate, please see Song, Storesletten and Zilibotti (2011). Therefore, growth is due to expansion of the more productive financially-unconnected firms at the expense of less productive financially-connected firms.

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Appendix: Proofs

Notice that FOCs (2) and (3) lead to the following two conditions

$$c_2 = (\delta (1+r) (1+\gamma))^{\frac{1}{\rho}} c_1$$
(11)

$$c_3 = (\delta (1+r))^{\frac{1}{\rho}} c_2.$$
(12)

If (2) does not hold, then c_2 is given by

$$c_2 = z_2 \times \left(1 + \frac{(\delta (1+r))^{\frac{1}{\rho}}}{1+r}\right)^{-1}.$$
 (13)

We also notice that when $U_R = U_D$, we must have either (i) $c_1^R < c_1^D$, $c_2^R > c_2^D$, and $c_2^R > c_2^D$ or (ii) $c_i^R = c_i^D$ for all *i*.

Proof of Lemma 1

To show that the lemma, it suffices to show that $\frac{\partial U_D}{\partial h} - \frac{\partial U_R}{\partial h} > 0$. There are only three possible cases to consider: (a) consumption smoothing is infeasible in both regimes; (b) it is feasible under delay but infeasible under reform; (c) it is feasible in both regimes. Consider case (a). Notice a change in h affects c_1^R but not c_2^R and c_3^R ; it also affects c_2^D and c_3^D but not c_1^D . It is easy to obtain

$$\frac{dU_R}{dh} = u'\left(c_1^R\right)\frac{-1}{1+\gamma}\tag{14}$$

and

$$\frac{\partial U_D}{\partial h} = u'\left(c_1^D\right) \underbrace{\overbrace{dc_1^D}^D}_{dh} + \delta u'\left(c_2^D\right) \frac{dc_2^D}{dh} + \delta^2 u'\left(c_3^D\right) \frac{dc_3^D}{dh} \quad (\because c_1 \text{ is independent of } h)$$

$$= \left(\delta u'\left(c_2^D\right) + \delta^2\left(\delta\left(1+r\right)\right)^{\frac{1}{\rho}} u'\left(c_3^D\right)\right) \frac{dc_2^D}{dh} \quad (\because (12))$$

$$= \left(1 + \frac{\left(\delta\left(1+r\right)\right)^{\frac{1}{\rho}}}{1+r}\right) \delta u'\left(c_2^D\right) \frac{dc_2^D}{dh} \quad (\because (3))$$

$$= \delta u'\left(c_2^D\right) \frac{dB}{dh}. \quad (\because (13) \text{ and setting } z_2 = B)$$

Therefore

$$\begin{aligned} \frac{\partial U_D}{\partial h} &- \frac{\partial U_R}{\partial h} &= u'\left(c_1^R\right) \frac{1}{1+\gamma} + \delta u'\left(c_2^D\right) \frac{dB}{dh} \\ &> u'\left(c_1^D\right) \frac{1}{1+\gamma} + \delta u'\left(c_2^D\right) \frac{dB}{dh} \quad (\because c_1^R < c_1^D) \\ &> \delta u'\left(c_2^D\right) \left(1+r\right) + \delta u'\left(c_2^D\right) \frac{dB}{dh} \\ &= \delta u'\left(c_2^D\right) \left(\left(1+r\right) + \frac{dB}{dh}\right) > 0. \end{aligned}$$

for all r > 0 because $dB/dh \ge -1$ as assumed in A1.

Case (b). Totally differentiate $U_D = u(c_1^D) + \delta u(c_2^D) + \delta^2 u(c_3^D)$ with respect to h, we obtain

$$\frac{\partial U_D}{\partial h} = \left(u'\left(c_1^D\right) + \delta u'\left(c_2^D\right)\left(\delta\left(1+r\right)\left(1+\gamma\right)\right)^{\frac{1}{\rho}} + \delta^2 u'\left(c_3^D\right)\left(\delta^2\left(1+r\right)^2\left(1+\gamma\right)\right)^{\frac{1}{\rho}} \right) \frac{dc_1^D}{dh} \\
= \frac{1}{\left(1+r\right)\left(1+\gamma+E\right)} \frac{dB}{dh} \qquad \text{(where } E \text{ is defined in } (5) \\
= \frac{u'\left(c_1^D\right)}{\left(1+\gamma\right)\left(1+r\right)} \frac{dB}{dh},$$

where the first line is due to (11) and (12), the second line due to (2) and (3), the third line due to collection of terms. Using (14), we obtain

$$\frac{\partial U_D}{\partial h} - \frac{\partial U_R}{\partial h} = \frac{u'\left(c_1^R\right)}{\left(1+\gamma\right)\left(1+r\right)} + \frac{u'\left(c_1^D\right)}{\left(1+\gamma\right)\left(1+r\right)}\frac{dB}{dh}$$
$$= \frac{1}{\left(1+\gamma\right)\left(1+r\right)}\left(u'\left(c_1^R\right) + u'\left(c_1^D\right)\frac{dB}{dh}\right)$$
$$\geq \frac{1}{\left(1+\gamma\right)\left(1+r\right)}\left(u'\left(c_1^R\right) - u'\left(c_1^D\right)\right) \quad (\because dB/dh \ge -1)$$

which is positive because $c_1^R < c_1^D$.

Case (c). In this case, since there is consumption smoothing in each regime, and equality of U_D and U_R also means equality of life-time incomes, i.e.,

$$y - h + \frac{y}{1+r} = x + \frac{B}{1+r}$$

Because the LHS decreases with h at a greater rate that the RHS does, it means that $\frac{\partial U_R}{\partial h} < \frac{\partial U_D}{\partial h}$.

Proof of Lemma 2

For regime j = R, D, suppose the consumption smoothing is feasible under the regime. Then a change in n affects all of c_1^j, c_2^j, c_3^j .

$$\begin{aligned} \frac{\partial U_{j}}{\partial n} &= A'(n) + u'\left(c_{1}^{j}\right)\frac{dc_{1}^{j}}{dn} + \delta u'\left(c_{2}^{j}\right)\frac{dc_{2}^{j}}{dn} + \delta^{2}u'\left(c_{3}^{j}\right)\frac{dc_{3}^{j}}{dn} \\ &= A'(n) + u'\left(c_{1}^{j}\right)\frac{dc_{1}^{j}}{dn} + \frac{u'\left(c_{1}^{j}\right)}{(1+\gamma)\left(1+r\right)}\frac{dc_{2}^{j}}{dn} + \frac{u'\left(c_{1}^{j}\right)}{(1+\gamma)\left(1+r\right)^{2}}\frac{dc_{3}^{j}}{dn} \qquad (\because (2) \text{ and } (3)) \\ &= A'(n) + \frac{u'\left(c_{1}^{j}\right)}{(1+\gamma)}\left((1+\gamma)\frac{dc_{1}^{j}}{dn} + \frac{1}{1+r}\frac{dc_{2}^{j}}{dn} + \frac{1}{(1+r)^{2}}\frac{dc_{3}^{j}}{dn}\right) \\ &= A'(n) + \frac{u'\left(c_{1}^{j}\right)}{1+\gamma}\left(-c_{1}^{j}\gamma'(n)\right), \end{aligned}$$

where the last step is due to differentiation of the budget constraint with respect to n.

Proof of Proposition 1

There are three cases to consider: (a) consumption smoothing is infeasible in both regimes; (b) it is feasible under delay but infeasible under reform; (c) it is feasible in both regimes. Consider case (a). A change in n affects only c_1^R and c_1^D , but not any of c_2^R , c_3^R , c_2^D , and c_3^D . More specifically,

$$\frac{\partial U_D}{\partial n} = A\left(n\right) + \frac{d}{dn}u\left(c_1^D\right),$$

where the second term in the RHS equals

$$\frac{d}{dn}u\left(\frac{x}{1+\gamma}\right) = -u'\left(\frac{x}{1+\gamma}\right)\frac{x\gamma'(n)}{(1+\gamma)^2} = -(1-\rho)u\left(\frac{x}{1+\gamma}\right)\frac{\gamma'(n)}{1+\gamma}$$

and

$$\frac{\partial U_R}{\partial n} = A\left(n\right) + \frac{d}{dn}u\left(c_1^R\right),$$

where the second term in the RHS equals

$$\frac{d}{dn}u\left(\frac{y-h}{1+\gamma}\right) = -u'\left(\frac{y-h}{1+\gamma}\right)\frac{(y-h)\gamma'(n)}{(1+\gamma)^2} = (1-\rho)u\left(\frac{y-h}{1+\gamma}\right)\frac{\gamma'(n)}{1+\gamma}.$$

Therefore

$$\frac{\partial U_D}{\partial n} - \frac{\partial U_R}{\partial n} = (1 - \rho) \left(u \left(\frac{y - h}{1 + \gamma} \right) - u \left(\frac{x}{1 + \gamma} \right) \right) \frac{\gamma'(n)}{1 + \gamma}$$

Since $u\left(\frac{y-h}{1+\gamma}\right) - u\left(\frac{x}{1+\gamma}\right) < 0$, the above expression is positive if $\rho > 1$ and is negative if $\rho < 1$. Making use of Lemma 1.3, we obtain the result that $\partial h^* / \partial n < 0$ if $\rho > 1$ and $\partial h^* / \partial n > 0$ if $\rho < 1$.

Case (b). Given that consumption smoothing is infeasible under reform, $\frac{\partial U_R}{\partial n}$ remains to be the same as found in case (a). That is,

$$\frac{\partial U_R}{\partial n} = A(n) - (1-\rho) u\left(\frac{y-h}{1+\gamma}\right) \frac{\gamma'(n)}{1+\gamma}$$

For the delay regime, according to Lemma 2,

$$\frac{\partial U_D}{\partial n} = A(n) - u'(c_1^D) c_1^D \frac{\gamma'(n)}{1+\gamma}$$
$$= A(n) - (1-\rho) u(c_1^D) \frac{\gamma'(n)}{1+\gamma}.$$

We thus obtain

$$\frac{\partial U_D}{\partial n} - \frac{\partial U_R}{\partial n} = (1 - \rho) \left(u \left(c_1^R \right) - u \left(c_1^D \right) \right) \frac{\gamma'(n)}{(1 + \gamma)}.$$

Since consumption smoothing is infeasible under reform, according to Lemma 1.2, $c_1^R < c_1^D$. As a result $\frac{\partial U_D}{\partial n} - \frac{\partial U_R}{\partial n}$ is positive if and only if $\rho > 1$. Making use of Lemma 1.3, we obtain the result that $\partial h^* / \partial n < 0$ if $\rho > 1$ and $\partial h^* / \partial n > 0$ if $\rho < 1$.

Case (c). Given that consumption smoothing is feasible in both regimes, $U_R = U_D$ if and only if the two life-time incomes are the same, i.e.,

$$y - h + \frac{y}{1+r} = x + \frac{B}{1+r}.$$

An infinitessimal increase in n has no effect on the fact that consumption smoothing is feasible in both regimes. It does not affect the equality of the two life-time incomes. As a result, h^* is independent of n.

Proof of Lemma 3

We use U_D^B to denote the utility under delay given post-delay income function B. Then it is clear that, for h < y - x, we have $U_D^{B=x} > U_D^{B=y-h}$. Because both U_R and U_D are decreasing in h, it must be the case that the <u>h</u> at which $U_D^{B=y-h}$ will intersect with U_R is smaller than the \overline{h} at which $U_D^{B=x}$ intersect with U_R .

Proof of Proposition 2 (alternative formulation of child rearing cost)

The proof consists of the following three steps:

- 1. Provided that y x < h, the "disposable" income profile under reform is more backloaded;
- 2. Given that $U_R = U_D$, either $c_1^R < c_1^D$ or $c_1^R = c_1^D$;
- 3. For j = R, D, whether or not consumption smoothing is feasible, we have

$$\frac{dU_{j}}{dn} = -u'\left(c_{1}^{j}\right)\left(\kappa'\left(n\right)z_{1}^{j} + T'\left(n\right)\right)$$

where $z_1^R = y$ and $z_1^D = x$.

4. For $j = R, D, h^*(n)$ is decreasing in n (even if consumption smoothing is feasible under both regimes and c1R = c1D)

Step 1: Under reform, the period-1 disposable income is $(1 - \kappa(n))y - T - h$ and the period-2 disposable income is y. Under delay, the period-1 disposable income is $(1 - \kappa(n))x - T$ and the period-2 disposable income is B(h). The disposable income profile is more back-loaded under reform if and only if

$$\frac{\left(1-\kappa\left(n\right)\right)y-T\left(n\right)-h}{y} < \frac{\left(1-\kappa\left(n\right)\right)x-T\left(n\right)}{B\left(h\right)}.$$

Because y > B(h), a sufficient condition for the equation to hold is $(1 - \kappa(n)) y - T(n) - h < (1 - \kappa(n)) x - T(n)$, i.e., $(1 - \kappa(n)) (y - x) < h$. A sufficient condition for the latter to hold is y - x < h.

Step 2: $U_R = U_D$ if and only if the disposable income profile is more back-loaded in reform than in delay. Hence, $U_R = U_D$ means one of the following holds true: (a) consumption smoothing is infeasible under both reform and delay; (b) consumption smoothing is infeasible under reform and feasible under delay; and (c) consumption smoothing is feasible under both reform and delay.

In case c), $U_R = U_D$ is achieved when $c_i^R = c_i^D$ for all *i*. In case a), $c_1^R = (1 - \kappa(n)) y - h$ and $c_1^D = (1 - \kappa(n)) x$. Then $c_1^R < c_1^D \Leftrightarrow (1 - \kappa(n)) y - h < (1 - \kappa(n)) x \Leftrightarrow (1 - \kappa(n)) (y - x) < h$, which is the case. Therefore, in this case, $c_1^R < c_1^D$ and $c_2^R > c_2^D$ and $c_3^R > c_3^D$. In case b) we argue that $c_1^R < c_1^D$. If not, either $c_1^R = c_1^D$ or $c_1^R > c_1^D$. In the former case, it must be that $u(c_1^R) = u(c_1^D)$, $u(c_2^R) = u(c_2^D)$, and $u(c_3^R) = u(c_3^D)$ but then consumption smoothing under reform is also feasible. A contradiction. In the latter case, it must be the case that $u(c_1^R) > u(c_1^D)$ and $u(c_2^R) < u(c_2^D)$, implying that

$$\begin{array}{lll} u'\left(c_{1}^{R}\right) & < & u'\left(c_{1}^{D}\right) \\ \\ u'\left(c_{2}^{R}\right) & > & u'\left(c_{2}^{D}\right). \end{array}$$

However, given consumption smoothing is feasible under delay, we have

$$u'\left(c_{1}^{D}\right) = \delta\left(1+r\right)u'\left(c_{2}^{D}\right).$$

using the above two relations, we have

$$u'\left(c_{1}^{R}\right) < \delta\left(1+r\right)u'\left(c_{2}^{R}\right),$$

which is a contradiction to the claim that under reform consumption smoothing is infeasible (meaning that $u'(c_1^R) \ge \delta(1+r) u'(c_2^R)$). Step 3: When consumption smoothing is infeasible under reform, $c_1^R = y - (T(n) + \kappa(n)y) - h = (1 - \kappa(n))y - T(n) - h$

$$\frac{dU_R}{dn} = -u'\left(c_1^R\right)\left(\kappa'\left(n\right)y + T'\left(n\right)\right)$$

When consumption smoothing is infeasible under delay, $c_{1}^{D} = x - (T(n) + \kappa(n)x)$

$$\frac{dU_{D}}{dn} = -u'\left(c_{1}^{D}\right)\left(\kappa'\left(n\right)x + T'\left(n\right)\right)$$

We next consider the case where consumption smoothing is feasible under regime j = R, D. The consumer maximization problem is to maximize

$$\max u\left(c_{1}^{j}\right) + \delta u\left(c_{2}^{j}\right) + \delta^{2} u\left(c_{3}^{j}\right)$$

by choosing c_1^j, c_2^j , and c_3^j subject to

$$c_{1}^{j} + \frac{c_{2}^{j}}{1+r} + \frac{c_{3}^{j}}{\left(1+r\right)^{2}} = \left(1-\kappa\left(n\right)\right)z_{1}^{j} - T\left(n\right) - \Delta_{j}h + \frac{z_{2}^{j}}{1+r}$$

Lagrangian:

$$L = u\left(c_{1}^{j}\right) + \delta u\left(c_{2}^{j}\right) + \delta^{2} u\left(c_{3}^{j}\right) \\ + \lambda\left(\left(1 - \kappa\left(n\right)\right)z_{1}^{j} - T\left(n\right) - \Delta_{j}h + \frac{z_{2}^{j}}{1 + r} - \left(c_{1}^{j} + \frac{c_{2}^{j}}{1 + r} + \frac{c_{3}^{j}}{\left(1 + r\right)^{2}}\right)\right)$$

where $z_1^R = y, \, \Delta_R = 1, z_2^R = y, z_1^D = x, \Delta_D = 0, z_2^D = B(h)$. FOCs

$$\begin{aligned} u'\left(c_{1}^{j}\right)-\lambda &= 0\\ \delta u'\left(c_{2}^{j}\right)-\frac{\lambda}{1+r} &= 0\\ \delta^{2}u'\left(c_{3}^{j}\right)-\frac{\lambda}{\left(1+r\right)^{2}} &= 0 \end{aligned}$$

Hence,

$$\begin{aligned} u'\left(c_{1}^{j}\right) &= \delta\left(1+r\right)u'\left(c_{2}^{j}\right) = \delta^{2}\left(1+r\right)^{2}u'\left(c_{3}^{j}\right) \\ c_{2}^{j} &= \left(\delta\left(1+r\right)\right)^{\frac{1}{\rho}}c_{1}^{j} \\ c_{3}^{j} &= \left(\delta^{2}\left(1+r\right)^{2}\right)^{\frac{1}{\rho}}c_{1}^{j} \end{aligned}$$

hence

$$\frac{d}{dn}U_{j} = u'\left(c_{1}^{j}\right)\frac{dc_{1}^{j}}{dn} + \delta u'\left(c_{2}^{j}\right)\frac{dc_{2}^{j}}{dn} + \delta^{2}u'\left(c_{3}^{j}\right)\frac{dc_{3}^{j}}{dn}$$
$$= u'\left(c_{1}^{j}\right)\frac{dc_{1}^{j}}{dn} + \frac{u'\left(c_{1}^{j}\right)}{1+r}\frac{dc_{2}^{j}}{dn} + \frac{u'\left(c_{1}^{j}\right)}{(1+r)^{2}}\frac{dc_{3}^{j}}{dn}$$
$$= u'\left(c_{1}^{j}\right)\left(\frac{dc_{1}^{j}}{dn} + \frac{1}{1+r}\frac{dc_{2}^{j}}{dn} + \frac{1}{(1+r)^{2}}\frac{dc_{3}^{j}}{dn}\right)$$

Differentiating the budget constraint, we obtain

$$\frac{d}{dn}\left(c_{1}^{j} + \frac{c_{2}^{j}}{1+r} + \frac{c_{3}^{j}}{\left(1+r\right)^{2}}\right) = -\kappa'(n) z_{1}^{j} - T'(n)$$

substituting this into the last equation, we obtain

$$\frac{dU_{D}}{dn} = u'\left(c_{1}^{j}\right)\left(-\kappa'\left(n\right)z_{1}^{j} - T'\left(n\right)\right),$$

where $z_1^R = y$ and $z_1^D = x$.

Step 4:

Hence

$$\frac{dU_D}{dn} - \frac{dU_R}{dn} = u'(c_1^R)(\kappa'(n)y + T'(n)) - u'(c_1^D)(\kappa'(n)x + T'(n)) \\
= \kappa'(n)(u'(c_1^R)y - u'(c_1^D)x) + T'(n)(u'(c_1^R) - u'(c_1^D)) \\
= \underbrace{\kappa'(n)u'(c_1^R)(y - x)}_{>0} + \underbrace{\kappa'(n)((u'(c_1^R) - u'(c_1^D))x)}_{\ge 0} + \underbrace{T'(n)(u'(c_1^R) - u'(c_1^D))x}_{\ge 0} + \underbrace{T'(n)(u'(c$$

Given that $c_1^R \leq c_1^D$ and y > x, the above equation must be strictly positive. This result, together with the property that $\frac{dU_R}{dh} - \frac{dU_D}{dh} < 0$ when $U_R = U_D$, implies that $dh^*/dn < 0$ because

$$\frac{dh^*}{dn} = \left(\frac{dU_D}{dn} - \frac{dU_R}{dn}\right) / \left(\frac{dU_R}{dh} - \frac{dU_D}{dh}\right).$$

Note that this result of $dh^*/dn < 0$ holds even if $c_1^R = c_1^D$ and consumption smoothing is feasible under both regimes. This is thus a stronger result that is Proposition 1.

Proof of Proposition 3 (aggregate decision making)

First we define our notations more formally. We use $P_{i\tau}$ to denote the population of stagei agents at period τ and n_{τ} the number of children for each young worker at period τ . Hence, population evolves according to $P_{1\tau} = n_{\tau-1}P_{2\tau}$. In addition, we make an explicit assumption that we are using the SM decision rule so that the hardship borne by each young agent when undergoing reform is the same, equal to k (although we still keep the notation of h, invariant over time). We omit the first case $(n_{\tau} < \alpha)$ which is most straightforward. We use h_t^m to denote the MEGEH in period t. For the other two cases, we first notice the following result:

Claim 1 Suppose $n^* \ge \alpha$. For any $\tau \ge t+1$, the MEGEH in period τ is $\overline{h}(n^*)$.

Proof. For any $\tau \ge t + 1$, $P_{1\tau} = n_{\tau-1}P_{2\tau} = n^*P_{2\tau} \ge \alpha P_{2\tau}$. Therefore, the currently young workers can overwhelm the currently middle aged workers. If the 1τ agents hold the belief that the reform, if not accepted now, will be accepted in the next period $(\tau + 1)$, then their greatest endurable hardship is simply $\overline{h}(n^*)$. By definition, it is impossible to support an

even greater endurable hardship. This greatest endurable hardship can be supported as an equilibrium outcome because, due to stationarity, the $1\tau + i$ agents having belief that "if the reform is not accepted in the current period, it will be accepted in the next period" can be supported, where i = 1, 2, ... Hence, h_t^m is indeed equal to $\overline{h}(n^*)$.

Consider case iii where $\alpha < n^* \leq n^t$. At period t, if the currently young workers believes that if the reform is not passed now it will be passed next period, then their GEH is $\overline{h}(n_t)$. Since this belief is credible because this hardship lower than $h_{t+1}^m(h_{t+1}^m = \overline{h}(n^*))$ due to claim 1 and $\overline{h}(n^*) \geq \overline{h}(n_t)$ because $n^* \leq n^t$, h_t^m is indeed equal to $\overline{h}(n_t)$. Next we consider case ii: $\alpha \leq n^t < n^*$. At period t, if the currently young workers believes that if the reform is not passed now it will be passed next period, then h_t^m is equal to $\overline{h}(n_t)$. However, this belief is incredible because the hardship $\overline{h}(n_t)$ is too high to be acceptable for stage-1 agents in period t + 1 ($h_{t+1}^m = \overline{h}(n^*)$ due to claim 1 and $\overline{h}(n^*) < \overline{h}(n_t)$ because $n_t < n^*$). For the same reason, any hardship strictly greater than $\overline{h}(n^*)$ will be acceptable for stage-1 workers in period t, unless $\overline{h}(n^*) < \underline{h}(n_t)$. Noting that h_t^m is bound below by $\underline{h}(n_t)$, we conclude that h_t^m is equal to max $\{\overline{h}(n^*), \underline{h}(n_t)\}$. This completes the proof.